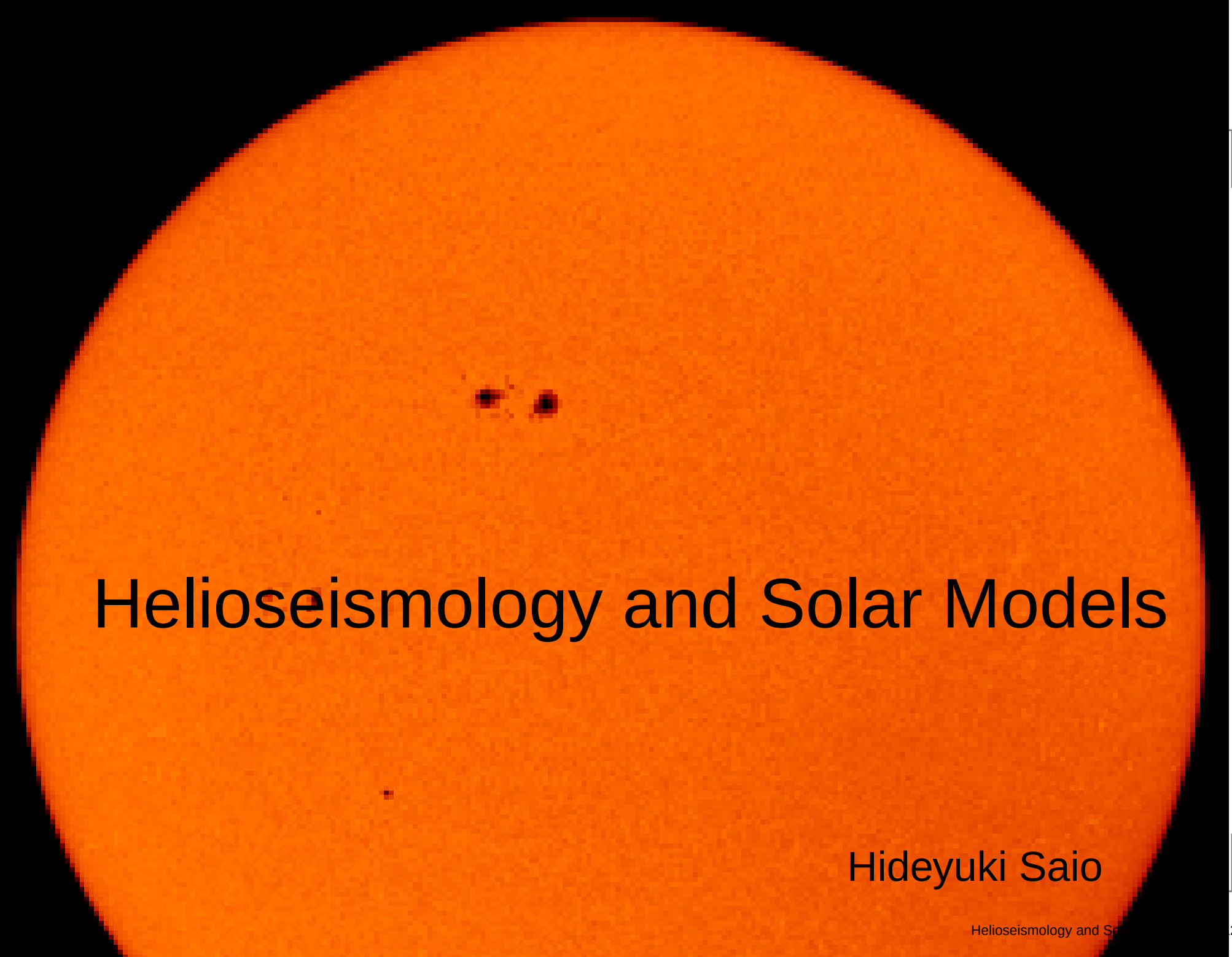


Helioseismology and Solar Models

Hideyuki Saio

Plan of my talk

- How to make a solar model
  - Basic equations
  - Evolution from the initial model to the present Sun
  - Internal structure of the Sun

- Helioseismology (‘Measuring’ solar internal structure via observed frequencies of Solar oscillations.)
  - Solar 5-mininute oscillations
  - Difference between the inferred structure (run of the sound speed) and solar models

How to make a solar model

Spherical symmetric and Hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho, \qquad \frac{dM_r}{dr} = 4\pi r^2 \rho \quad \left(M_r = 4\pi \int_0^r r^2 \rho dr \right)$$

Energy transport: Radiation or Convection

$$\frac{dT}{dr} = -\frac{3\kappa\rho F_{\rm rad}}{4acT^3} \quad \text{(Rad.); or} \quad \frac{dT}{dr} = \left(\frac{dT}{dP}\right)_{\rm ad} \frac{dP}{dr} \quad \text{(Con.)}$$

Energy conservation: $$\frac{d(r^2 F)}{r^2 dr} = \rho\epsilon_{\rm n} - \rho T \frac{dS}{dt}$$

Mass fraction X_i of element i:

$$\frac{\partial X_i}{\partial t} = \left(\frac{\partial X_i}{\partial t}\right)_{\rm nuc} + \frac{\partial}{\partial r}\left(D\frac{\partial X_i}{\partial r}\right); \qquad D = D_{\rm mol.dif} + D_{\rm rot} + D_{\rm conv}$$

Nuclear reactions ($T \gtrsim 10^7$K)

pp-chain

$$\mathrm{p} + \mathrm{p} \longrightarrow {}^2\mathrm{H} + \mathrm{e}^+ + \nu_\mathrm{e}$$

$$\mathrm{p} + {}^2\mathrm{H} \longrightarrow {}^3\mathrm{He} + \gamma$$

$$\begin{cases} {}^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2\mathrm{p} \\ \text{or} \\ {}^3\mathrm{He} + {}^4\mathrm{He} \longrightarrow {}^7\mathrm{Be} + \gamma \end{cases}$$

$$\begin{cases} {}^7\mathrm{Be} + \mathrm{e}^- \longrightarrow {}^7\mathrm{Li} + \nu_\mathrm{e} \\ {}^7\mathrm{Li} + \mathrm{p} \longrightarrow 2{}^4\mathrm{He} \end{cases}$$

or

$$\begin{cases} {}^7\mathrm{Be} + \mathrm{p} \longrightarrow {}^8\mathrm{B} + \gamma \\ {}^8\mathrm{B} \longrightarrow {}^8\mathrm{Be}^* + \mathrm{e}^+ + \nu_\mathrm{e} \\ {}^8\mathrm{Be}^* \longrightarrow 2{}^4\mathrm{He} \end{cases}$$

CNO Cycle

$$\underline{{}^{12}\mathrm{C}} + \mathrm{p} \longrightarrow {}^{13}\mathrm{N} + \gamma$$

$${}^{13}\mathrm{N} \longrightarrow {}^{13}\mathrm{C} + \mathrm{e}^+ + \nu_\mathrm{e}$$

$${}^{13}\mathrm{C} + \mathrm{p} \longrightarrow {}^{14}\mathrm{N} + \gamma$$

$$\underline{{}^{14}\mathrm{N}} + \mathrm{p} \longrightarrow {}^{15}\mathrm{O} + \gamma$$

$${}^{15}\mathrm{O} \longrightarrow {}^{15}\mathrm{N} + \mathrm{e}^+ + \nu_\mathrm{e}$$

$${}^{15}\mathrm{N} + \mathrm{p} \longrightarrow \begin{cases} \underline{{}^{12}\mathrm{C}} + {}^4\mathrm{He} \\ \text{or} \\ {}^{16}\mathrm{O} + \gamma \end{cases}$$

$${}^{16}\mathrm{O} + \mathrm{p} \longrightarrow {}^{17}\mathrm{F} + \gamma$$

$${}^{17}\mathrm{F} \longrightarrow {}^{17}\mathrm{O} + \mathrm{e}^+ + \nu_\mathrm{e}$$

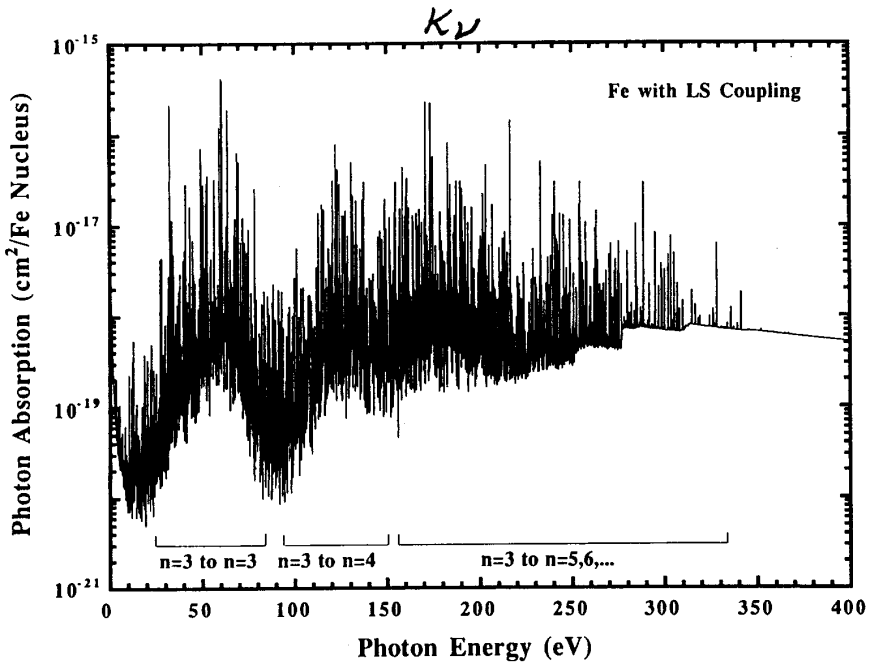
$${}^{17}\mathrm{O} + \mathrm{p} \longrightarrow \underline{{}^{14}\mathrm{N}} + {}^4\mathrm{He}$$

Black boxes for a model builder

- Radiative opacity κ:

$$\frac{1}{\kappa} = \frac{\int_0^\infty \kappa_\nu{}^{-1} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$

We depend on OPAL opacity tables, which give κ as a function of (ρ, T) for a given chemical composition of the gas.



- Nuclear reaction rates: $\epsilon_{\rm n} = \sum_{a,b} N_a N_b \langle \sigma_{ab} v \rangle Q_{ab}$

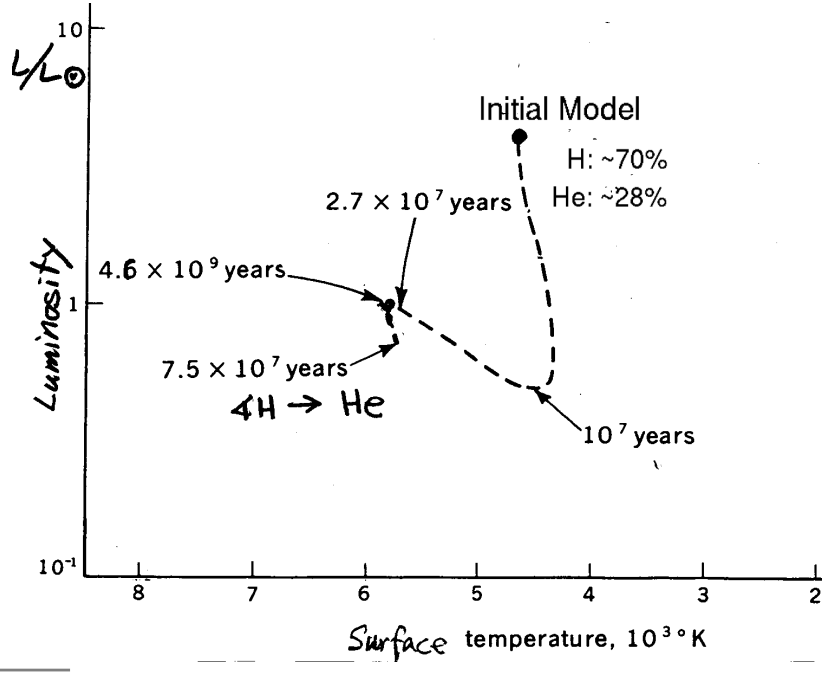
$\sigma_{ab} = S(0)_{ab} f(E) E^{-1} \exp(-2\pi Z_a Z_b e^2/\hbar v)$

We use some tables which list formulas $f(\rho, T)$ for $\langle \sigma_{ab} v \rangle$.

Initial model to the present Sun

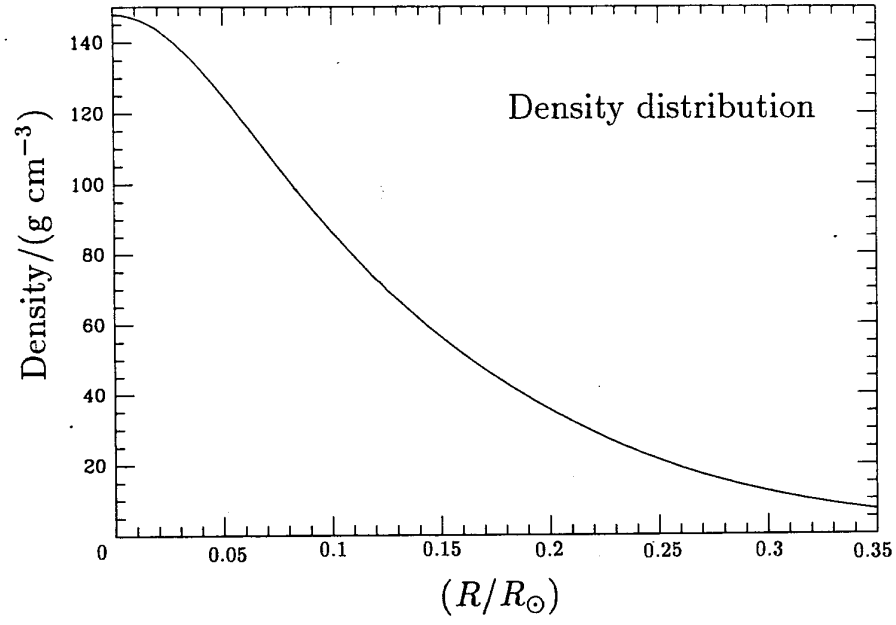
Initial model: $M = M_\odot = 1.99 \times 10^{33}$g

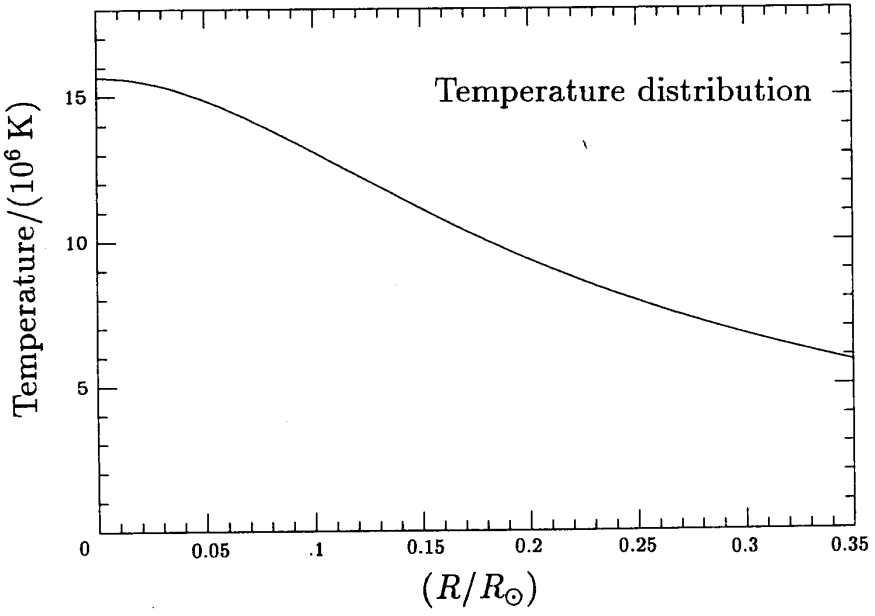
- $X_{\rm H}$ and $X_{\rm He}$ are homogeneous.
- $T_{\rm c} \ll 10^7$, No nuclear reaction; $L > L_\odot$. $R \gg R_\odot$
- The exact condition of the initial model is not important for the structure of the present Sun.

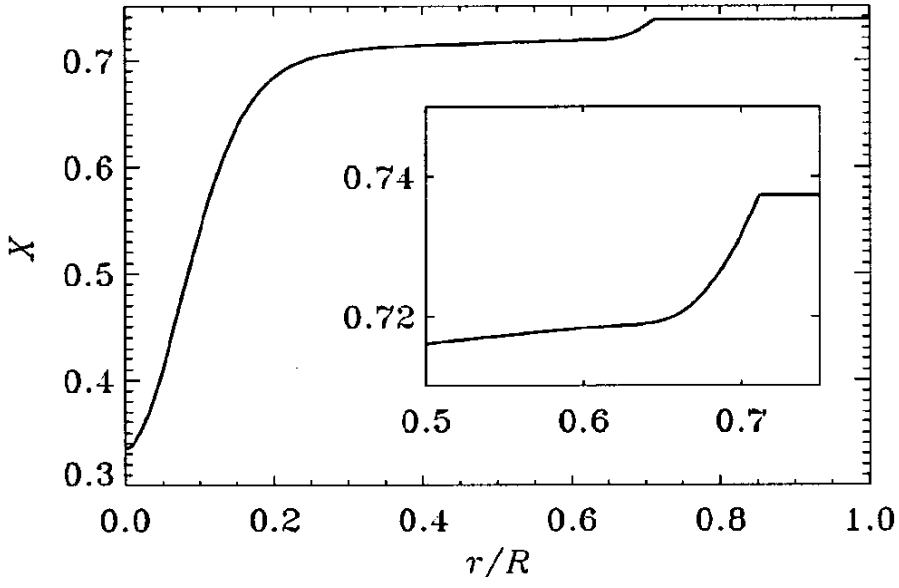


- A few hundred models are computed with sufficiently small Δt from the initial model to the present Sun.
- The age of the present Sun is 4.6×10^9yr.

Internal structure of the Sun







$\rho_c = 153$ g/cm^3

$T_c = 1.57 \times 10^7$ K

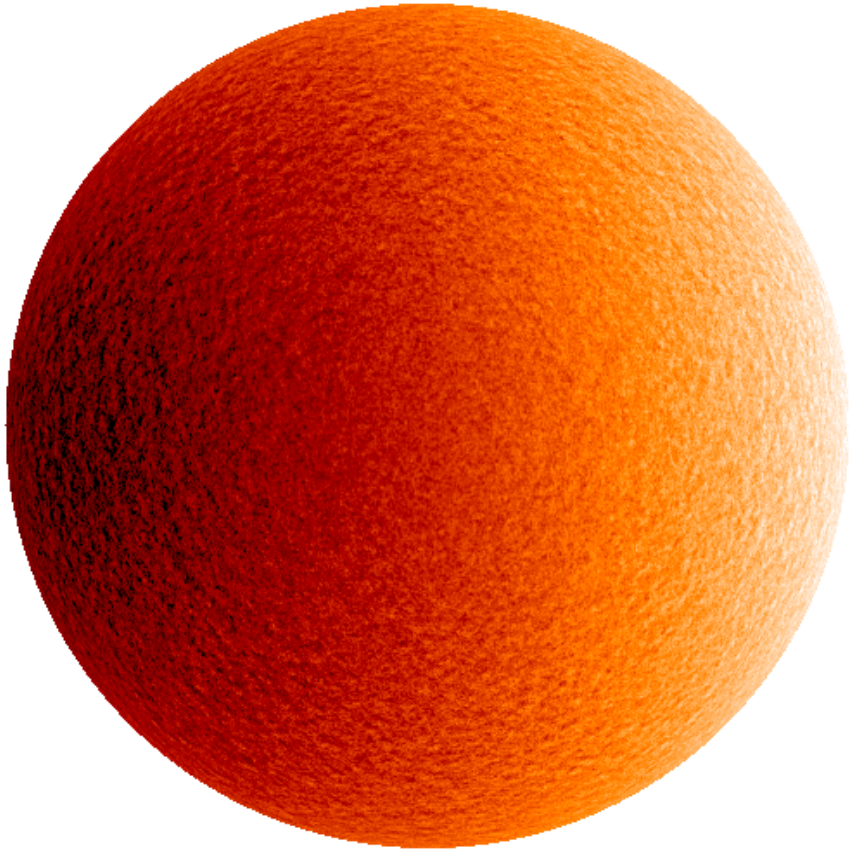
$X_c = 0.34, X_s \approx 0.74$(init. 0.71)

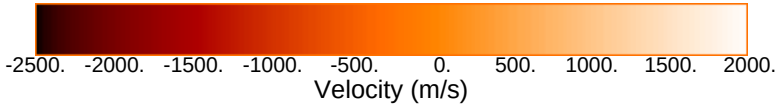
$$\left[\begin{array}{l} {}^{71}\mathrm{Ga(SNU)} = 127 \\ {}^{37}\mathrm{Cl(SNU)} = 7.25 \\ {}^{8}\mathrm{B} = 4.82 \times 10^6 \mathrm{cm}^{-2}\mathrm{s}^{-1} \end{array}\right.$$

Brun et al.(1999)

Velocity field on the surface of the Sun

Single Dopplergram
(30-MAR-96 19:54:00)





SOI / MDI

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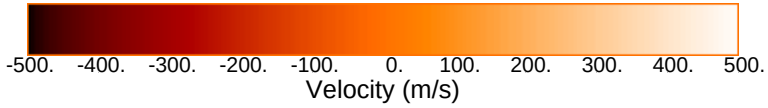
Single Dopplergram Minus 45 Images Average
(30-MAR-96 19:54:00)



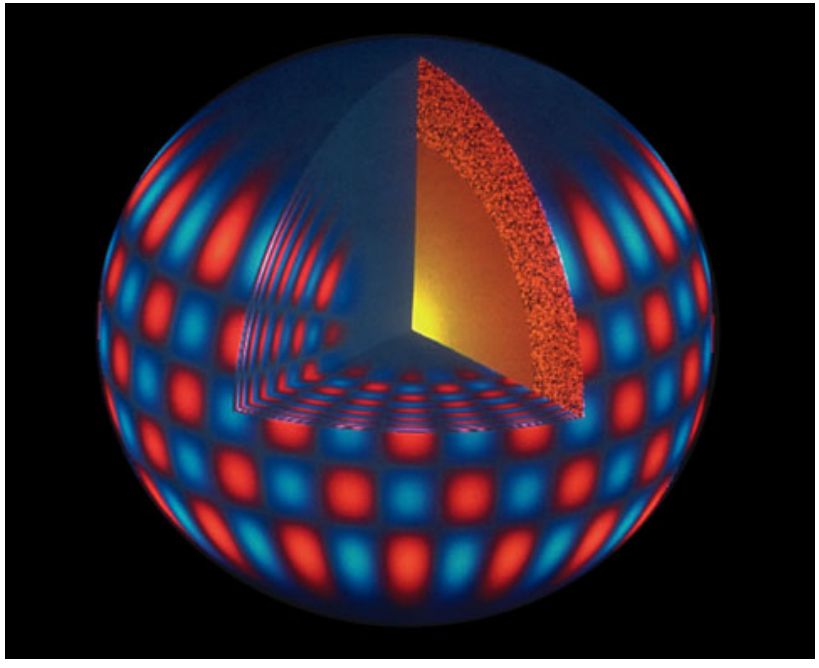
SOI / MDI

Stanford Lockheed Institute for Space Research

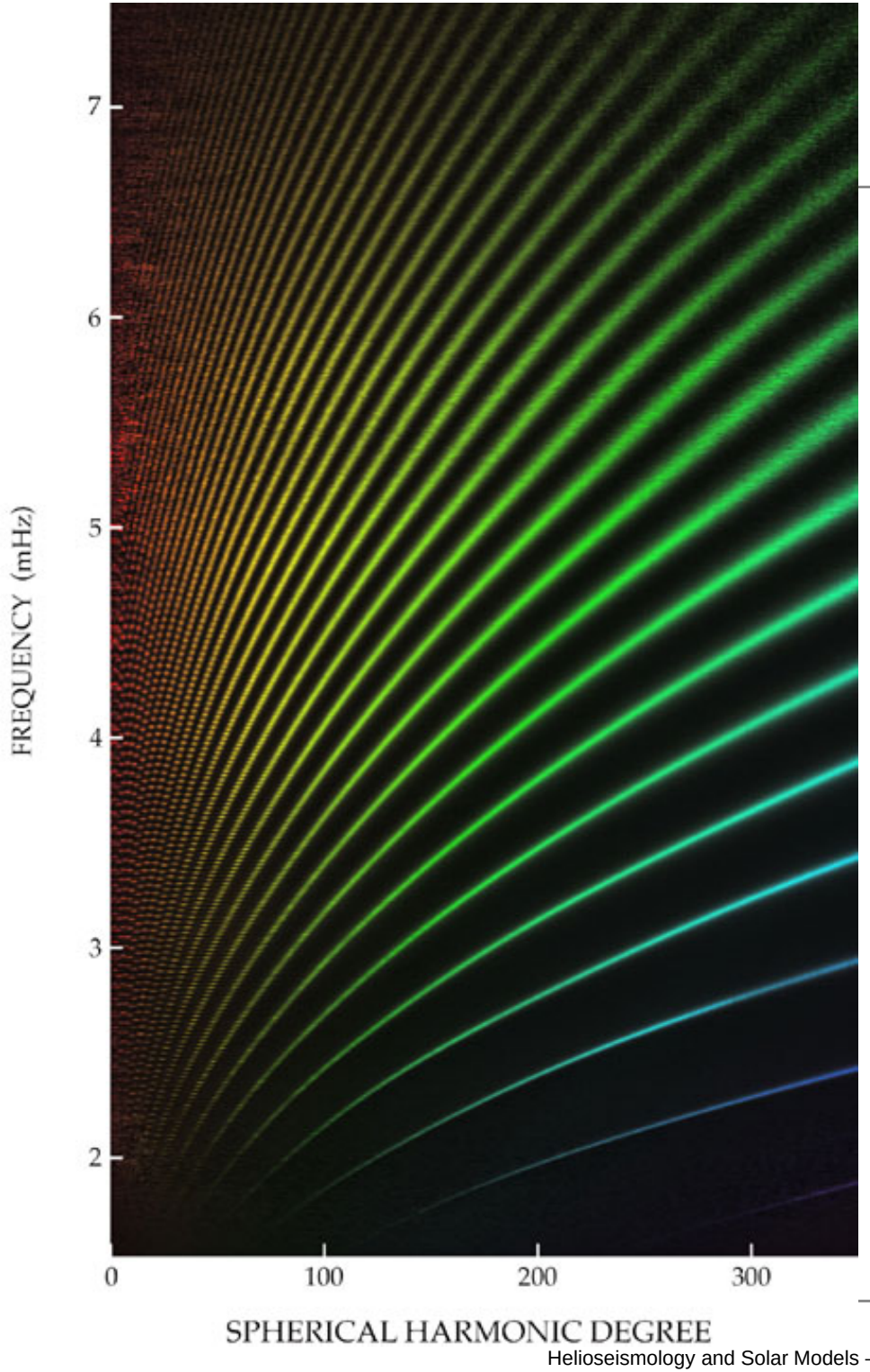
Solar and Heliospheric Observatory (SOHO) spacecraft

Fourier decomposition

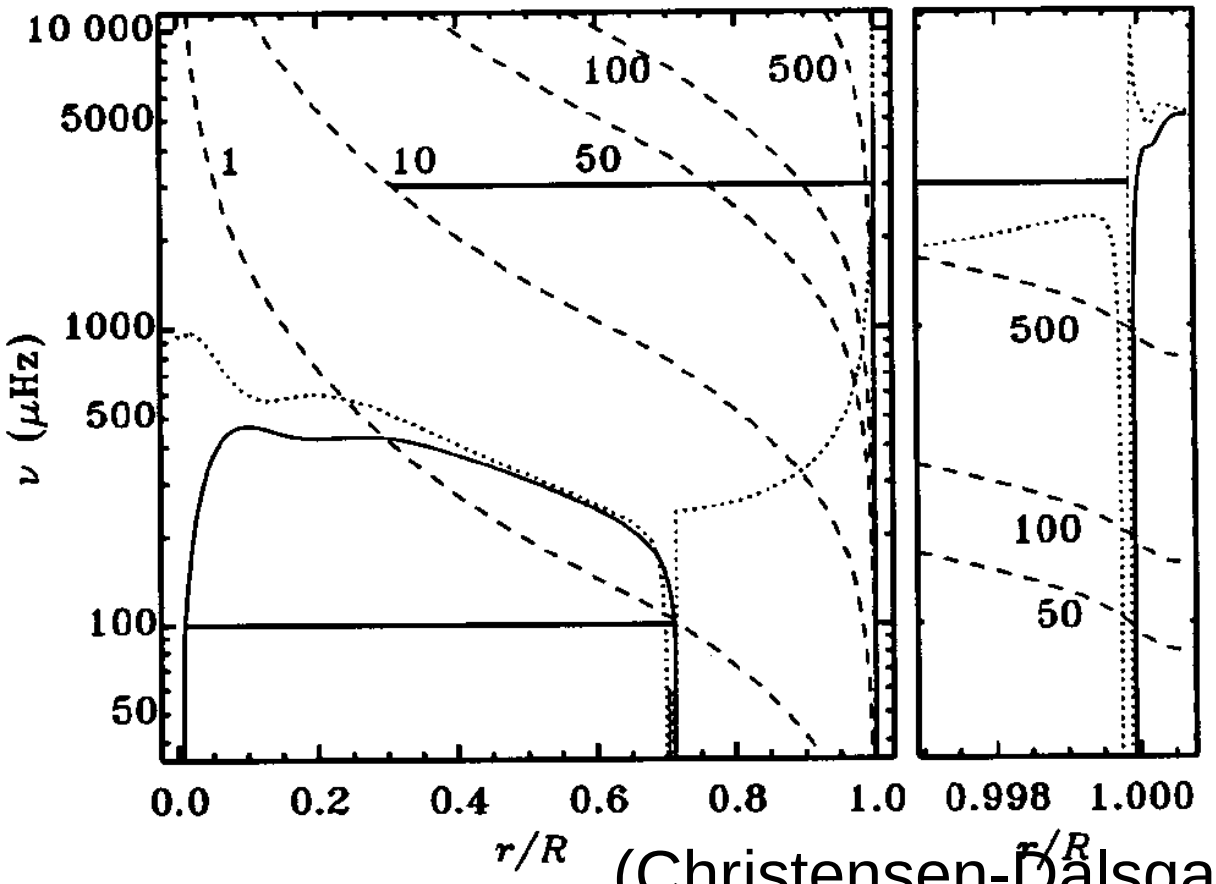
$$A_{\ell mn} Y_\ell^m(\theta, \phi) \exp(2i\pi\nu_{\ell mn} t)$$



A superposition of millions of normal modes of oscillation (resonating sound waves) of the Sun.



Inversion; frequencies $\longrightarrow c^2(r) \propto T/m$

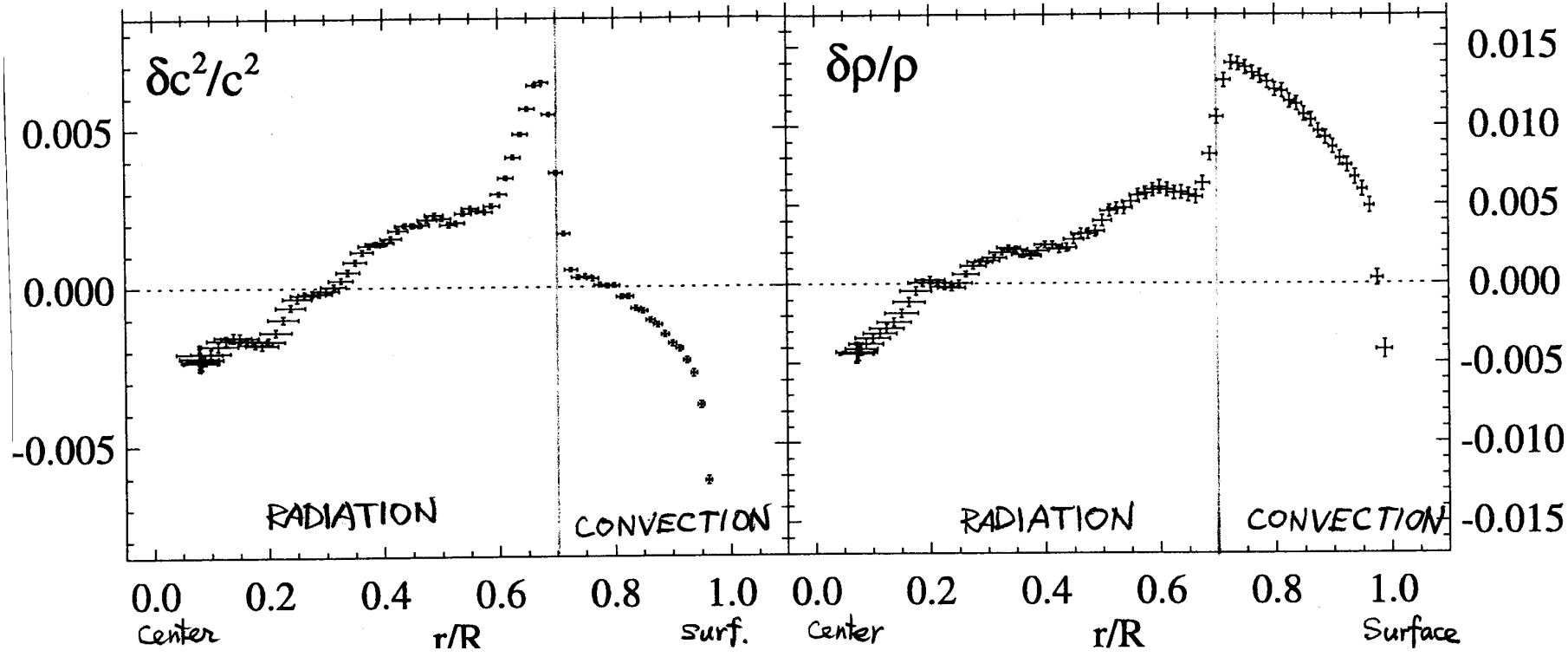


(Christensen-Dalsgaard 2002)

$$\int_{r_1}^{R} \sqrt{\omega^2 - \frac{\ell(\ell+1)c^2}{r^2}}\,\frac{dr}{c} = \pi(n+\alpha) \qquad (\omega \equiv 2\pi\nu)$$

Difference between Sun and Model

Sun – model (Brun et al.1999)



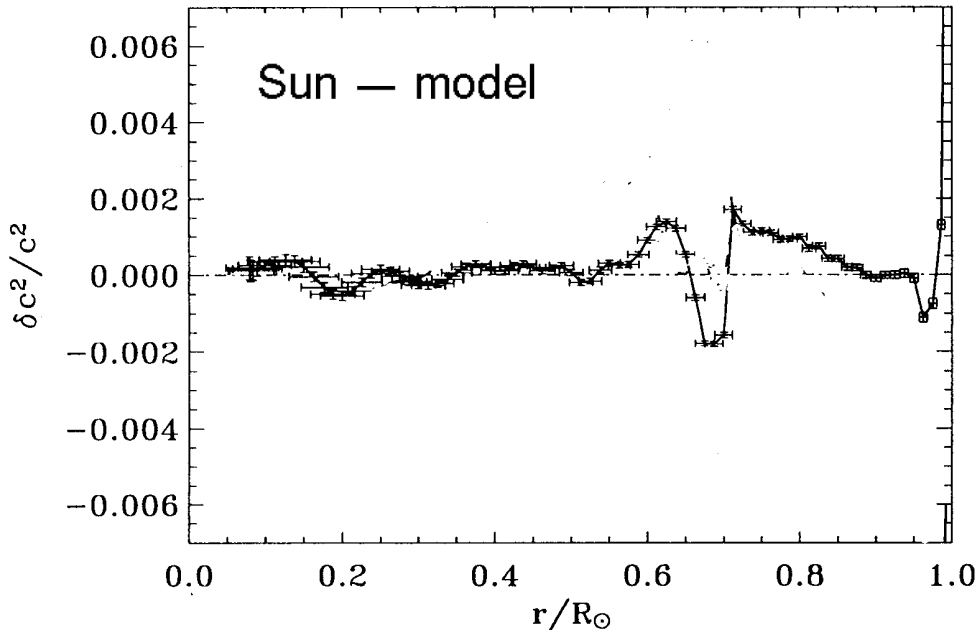
$$\frac{\delta c^2}{c^2} \longrightarrow \frac{\delta \rho}{\rho}$$

Spherical symmetry
Hydrostatic relation

Couvidat, Turck-Chiéze, & Kosovichev (2003)

Seismic model

1% increase of $S(0)_{\rm pp}$
+
3.5% increase of heavy-element abundance
↓
Seismic model
(Couvidat et al. 2003)



	SSM		Seismic model
$^{71}\mathrm{Ga(SNU)}$:	127	$\rightarrow$	128
$^{37}\mathrm{Cl(SNU)}$:	7.25	$\rightarrow$	7.48
$^{8}\mathrm{B}(10^6\mathrm{cm}^{-2}\mathrm{s}^{-1})$:	4.82	$\rightarrow$	4.98

SNO toal ^{8}B flux: $\phi(\nu_{\rm e}+\nu_{\mu\tau}) = (5.09 \pm 0.4) \times 10^6$ cm^{-2}s^{-1}