Nuclear liquid-gas phase transition in a molecular dynamics approach

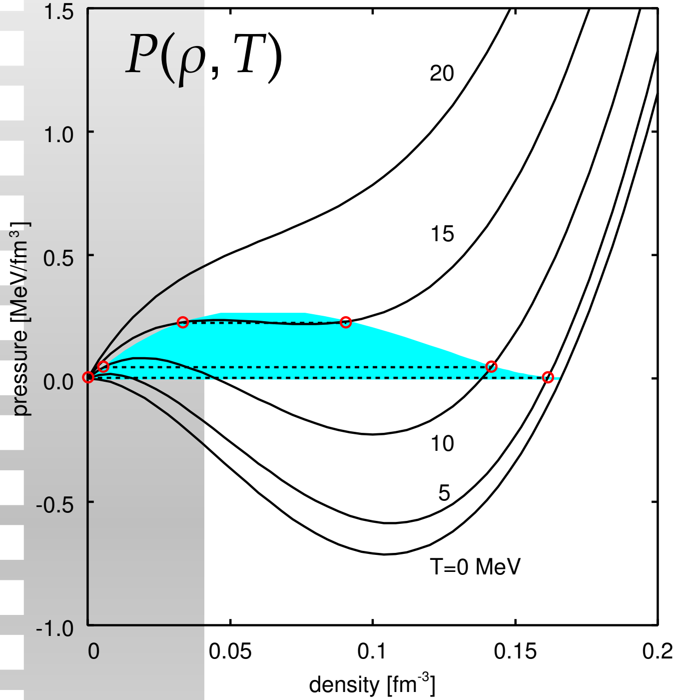
Akira Ono (Tohoku University)

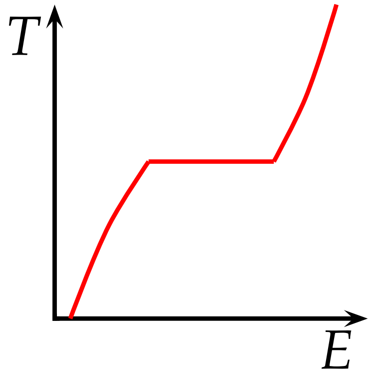
- Liquid-gas phase transition & Multifragmentation in nuclear collisions
- Antisymmetrized molecular dynamics (AMD) approeach
- Phase transition studied with AMD

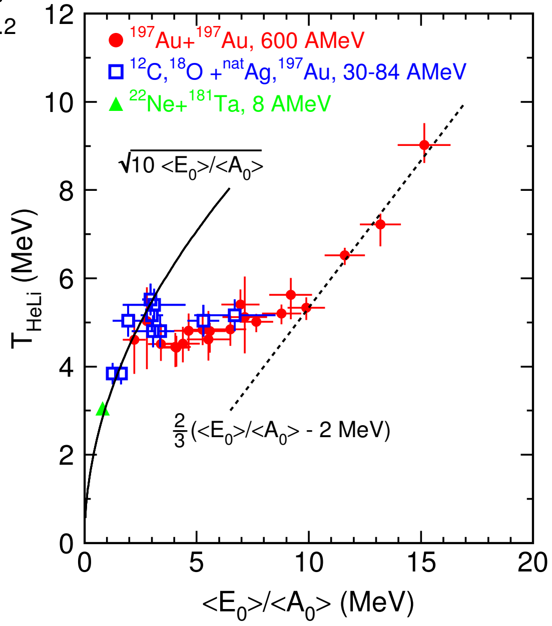
  T. Furuta & A.O., Phys. Rev. C, in press

Liquid-Gas Phase Transition

EOS of uniform nuclear matter (≈ Van der Waals)



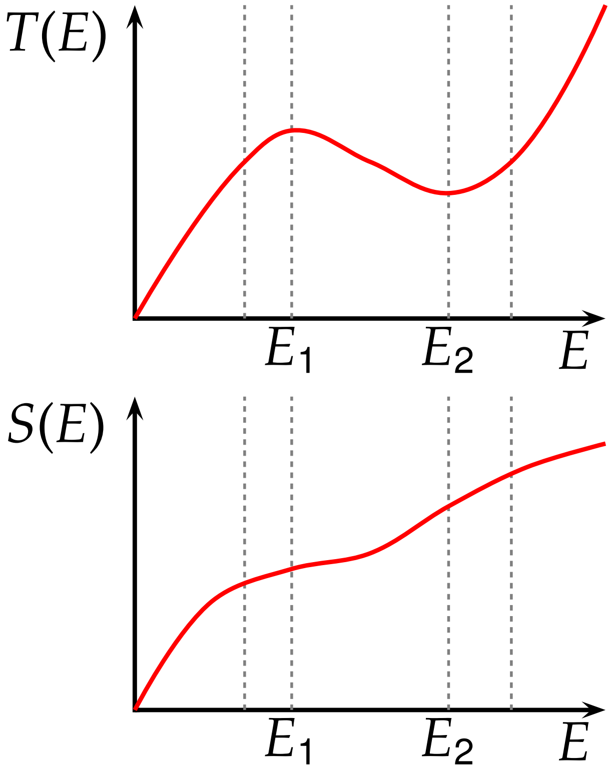




Pochodzalla et al., PRL75 (1995) 1040.

Finite systems

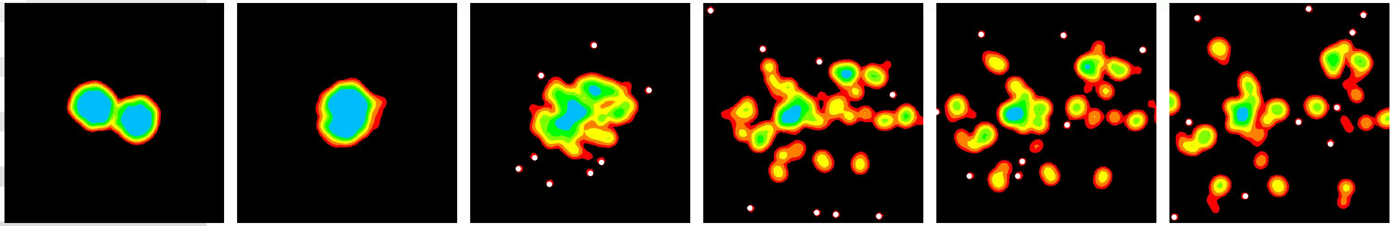
Microcanonical

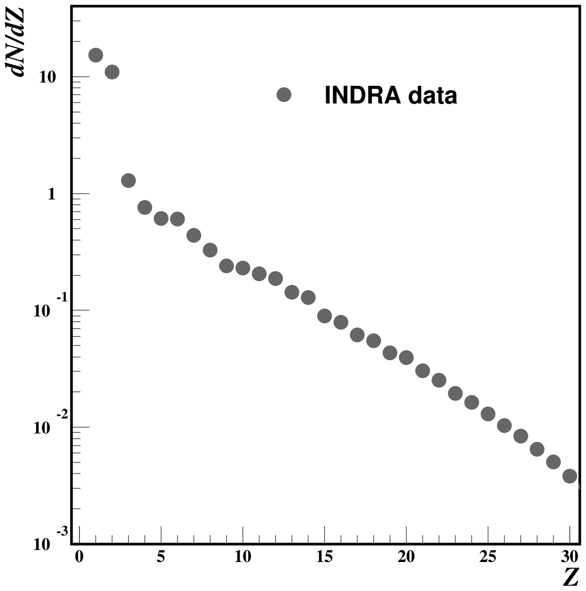


$$S(E) = \log W(E)$$

$$T(E) = \left(\frac{\partial S}{\partial E}\right)^{-1}$$

Multifragmentation





Fragment size distribution

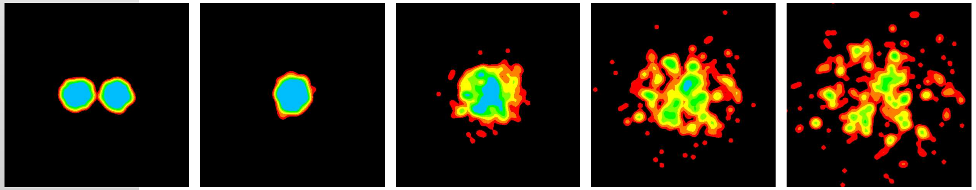
Xe + Sn, 50 MeV/nucleon

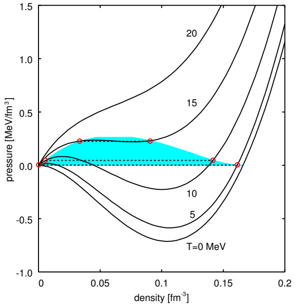
- Incident energy 50 MeV/nucleon
  $\Leftrightarrow$ Available energy 12.5 MeV/nucleon
  $>$ (B.E. $\approx$ 8 MeV/nucleon)
  However, most nucleons are bound in fragments.
- Excitation energy of fragments $\sim$ 3 MeV/nucleon $\ll E_F$
  Quantum descriptions are required.

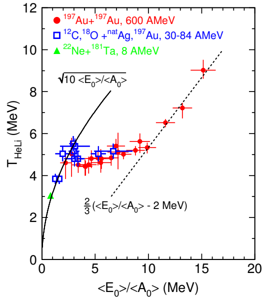
What are important for multifragmentation?

- Saturation property of nuclei (nuclear matter)
- Low density $\Leftarrow$ Collision dynamics
- Statistical (equilibrium) property — Liquid-gas phase transition

Dynamics and Equilibrium







Dynamical Aspect

Statistical Aspect

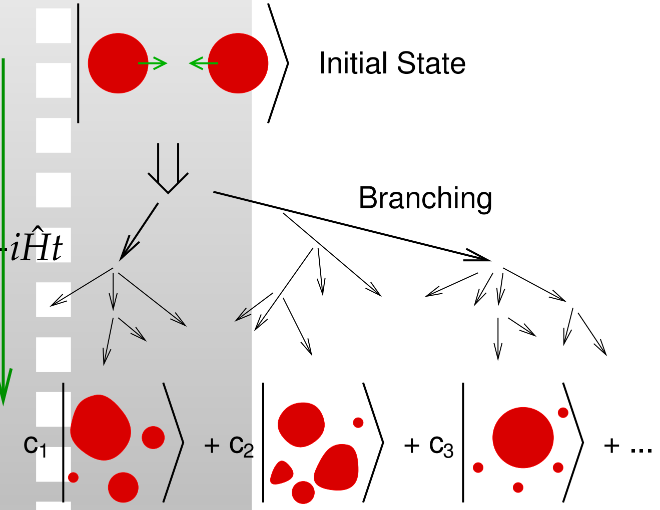
"Is equilibrium relevant in dynamical collisions?"

$\Rightarrow$ Need a unified description.

"Molecular dynamics" should describe both:

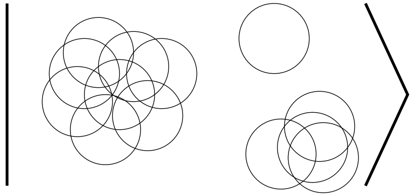
- Dynamical stage of the reactions
- Equilibrium, phase transition
- (Properties of nuclei)

Antisymmetrized Molecular Dynamics



AMD wave function

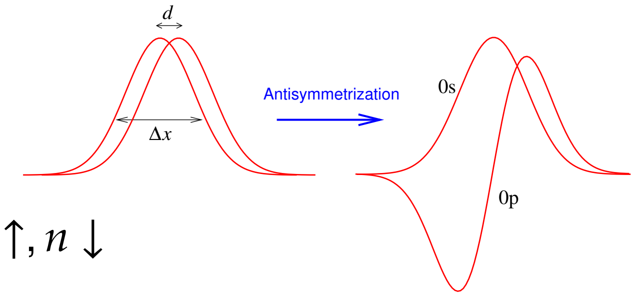
$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$



$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$



Stochastic equation of motion for the wave packet centroids Z:

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + \text{(NN collisions)} + \Delta\mathbf{Z}_i(t)$$

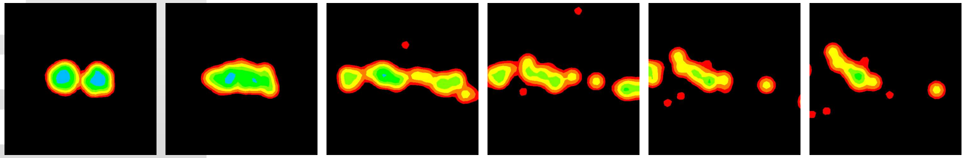
- Time evolution of single-particle wave functions in the mean field
- Nucleon-nucleon collisions (as the residual interaction)

Energy is conserved. No temperature in the equation.

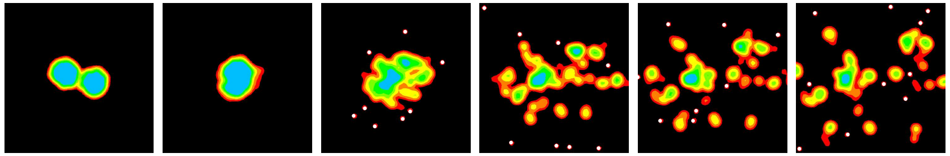
Quantum effects are included.

AMD results for fragmentation

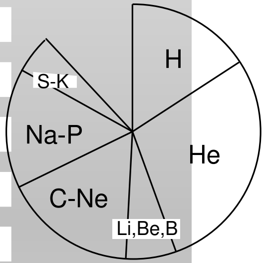
^{40}Ca + ^{40}Ca at 35 MeV/u, $b = 0$



Xe + Sn at 50 MeV/u, $0 \le b \le 4$ fm



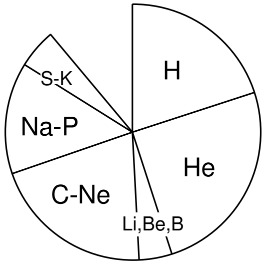
Experiment



Hagel et al.

PRC50(1994)2017

AMD

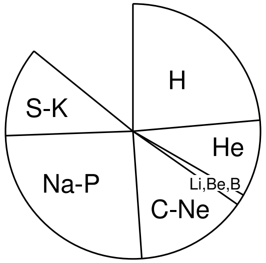


(Gogny force)

Soft EOS,

p-dep U

AMD

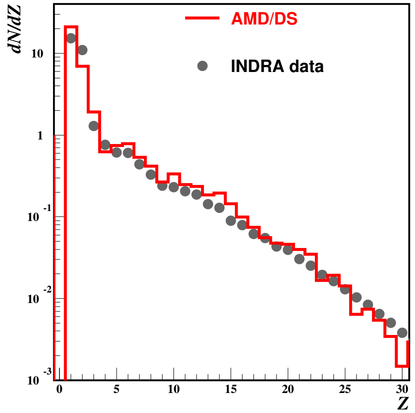


(SKG2 force)

Stiff EOS,

p-indep U

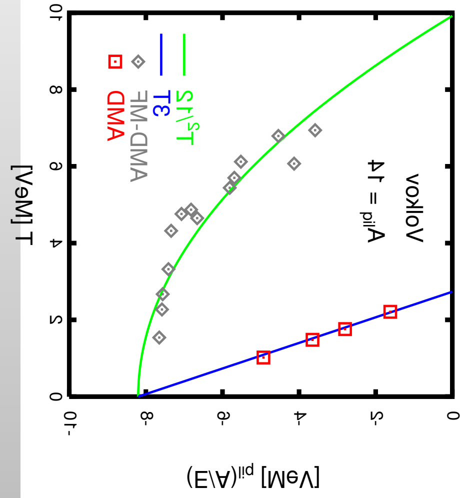
Charge distribution



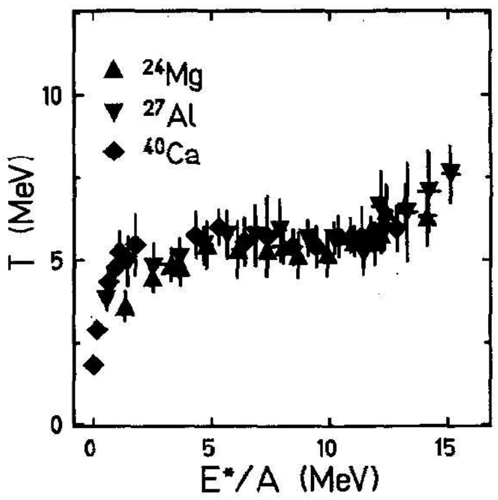
A.O. et al., Phys. Rev. C 66 (2002) 014603.

Caloric curves calculated with other MD models

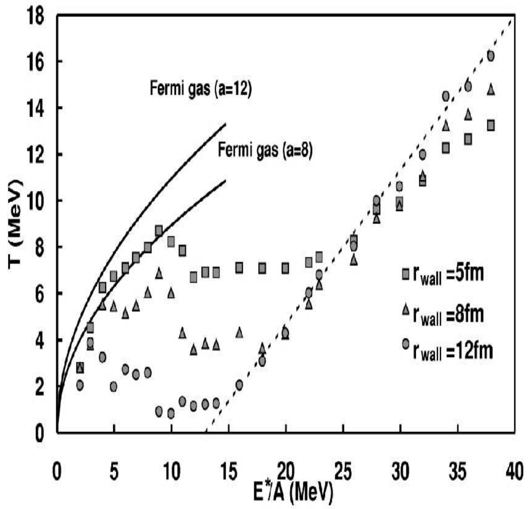
Ono & Horiuchi



Schnack & Feldmeier



Sugawa & Horiuchi



- Ono and Horiuchi, PRC53 (1996) 2341.
- J. Schnack and H. Feldmeier, PLB 409, 6 (1997).
- Y. Sugawa and H. Horiuchi, PRC 60, 064607 (1999); Prog. Theor. Phys. 105, 131 (2001).

These are not satisfactory because ...

- Not applicable to nuclear reactions.
- Not consistent with the nuclear matter saturation property.
- Pressure is not constant.

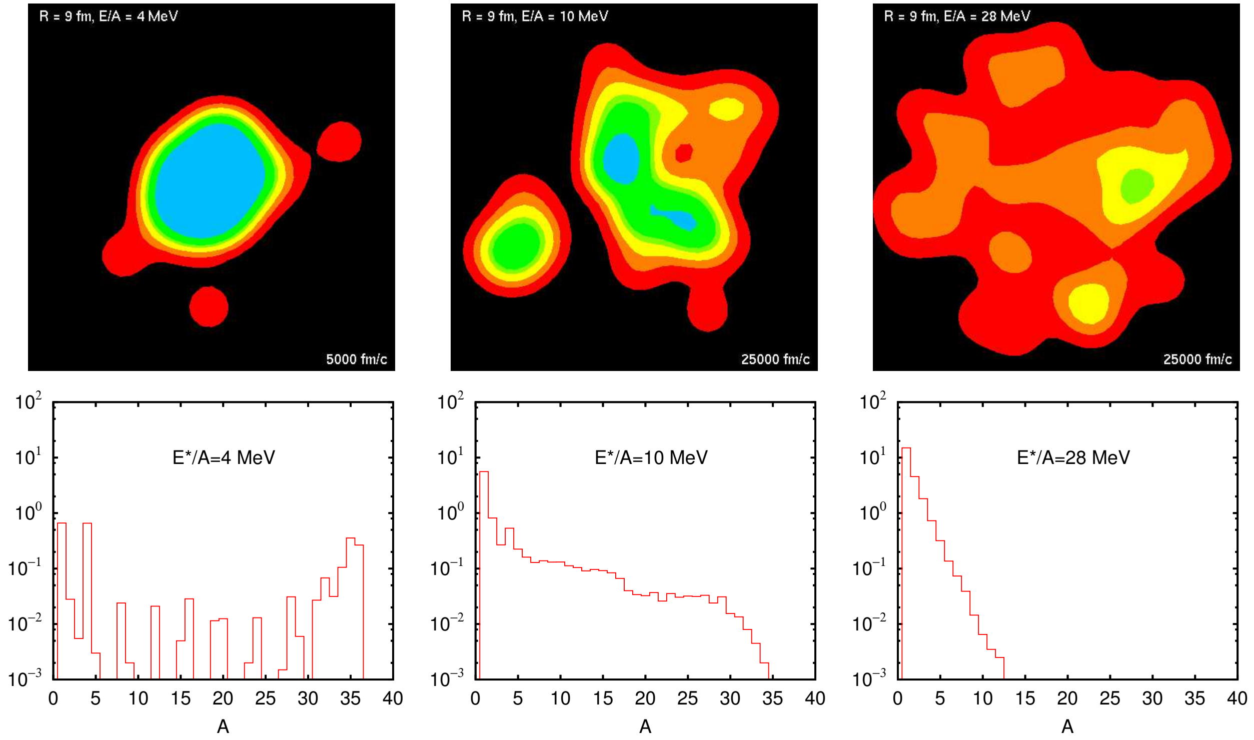
Microcanonical ensemble produced by AMD

Microcanonical ensemble ⇐ Simply solve the time evolution for a long time

- Total energy of the system: E
- Volume: $V = \frac{4}{3}\pi R^3$ (Reflection on the boundary)
- Neutron and protpon numbers: $N = 18$, $Z = 18$

$V = \frac{4}{3}\pi(9\ \mathrm{fm})^3$

$E^*/A = 4$ MeV $\quad$ $E^*/A = 10$ MeV $\quad$ $E^*/A = 28$ MeV



Temperature and Pressure

- Temperature of an ensemble ⇐ Gas-like nucleons

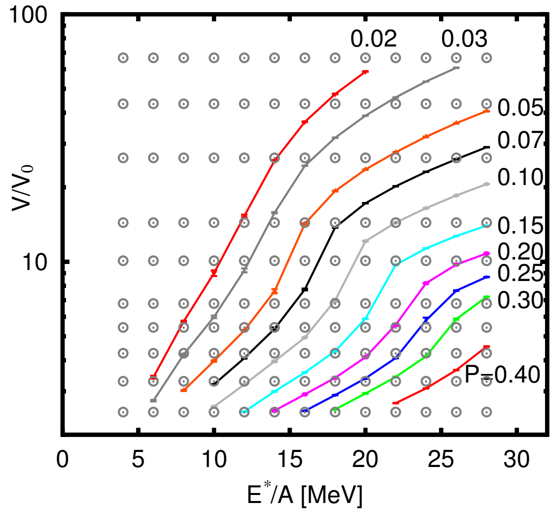
$$\frac{1}{T} = \frac{\partial S(E)}{\partial E} = \left\langle \frac{\partial S_{\text{gas}}(E_{\text{gas}})}{\partial E_{\text{gas}}} \right\rangle_E = \left\langle \frac{\frac{3}{2}N_{\text{gas}} - 1}{E_{\text{gas}}} \right\rangle_E \approx \frac{3}{2}\left\langle \frac{E_{\text{gas}}}{N_{\text{gas}}} \right\rangle_E^{-1}$$

Very stable against the change of the definition of gas-like nucleons.

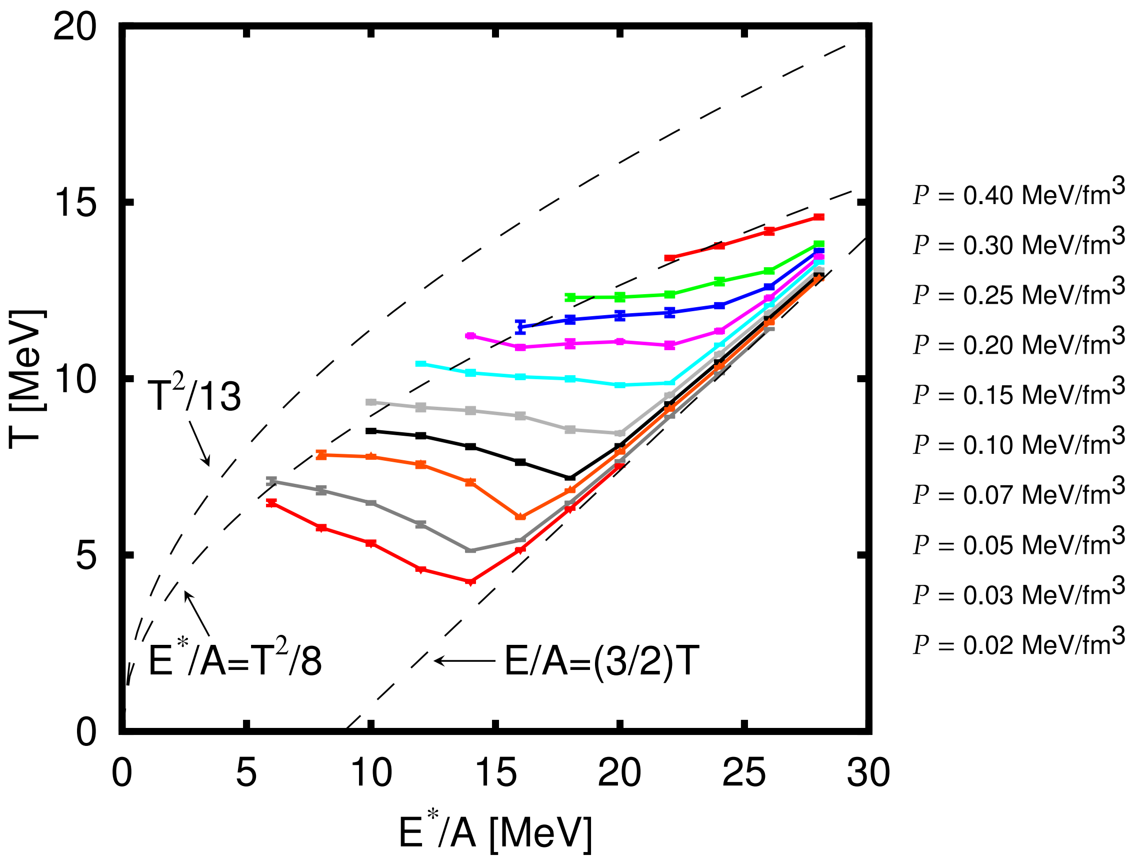
- Pressure of an ensemble ⇐ Reflections on the boundary

$$P = \frac{2\sum_{\text{reflections}} \Delta\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi R^2 \times (\text{time})}$$

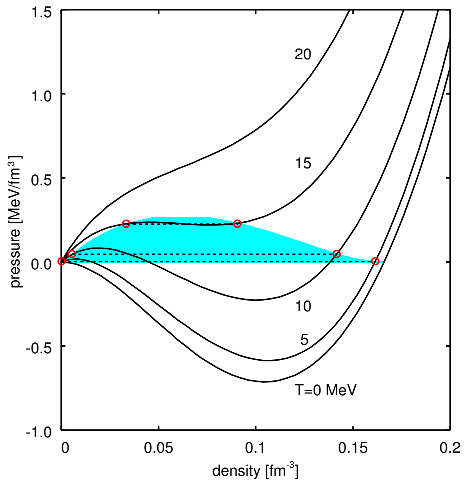
Curves of $P = \text{const.}$



Caloric curve by AMD



Infinite matter



- Negative heat capacity was obtained. (Phase transition)
- From liquid-gas coexistence to gas phase
- Consistent with the quantum relation $E^*_{\rm liq} = aT^2$ with $a = A/$(8-13 MeV).
- Critical point $(T_{\rm c}, P_{\rm c}) \approx$ (12 MeV, 0.2 MeV/fm^3)

Summary

- Dynamics and statistics in heavy-ion collisions
- AMD
  - Stochastic equation of motion $t \to t + \Delta t$
  - Applicable for $t = 0 \to$?00 fm/c (reactions)
  - Applicable for $t \to \infty$ (equilibrium)
- AMD is consistent with
  - the existence of the liquid-gas phase transition in nuclear many-body system.
  - the quantum statistical property of nuclear system.
- A unified description of dynamics and equilibrium is now possible.
  - How is eqilibirum relevant in dynamical reactions?
  - Other systems: $N \neq Z$, Large A, ...