Phase transition

in finite systems

Francesca GULMINELLI and Philippe CHOMAZ

- **Finite systems**
  - Zeroes of the partition sum
  - Bimodal event distributions
  - Negative heat capacity
  - Abnormal fluctuations
- **Experimental results**
  - Melting of Na clusters
  - Superfluidity in atomic nuclei
  - Fragmentation of nuclei
  - Fragmentation of H-clusters

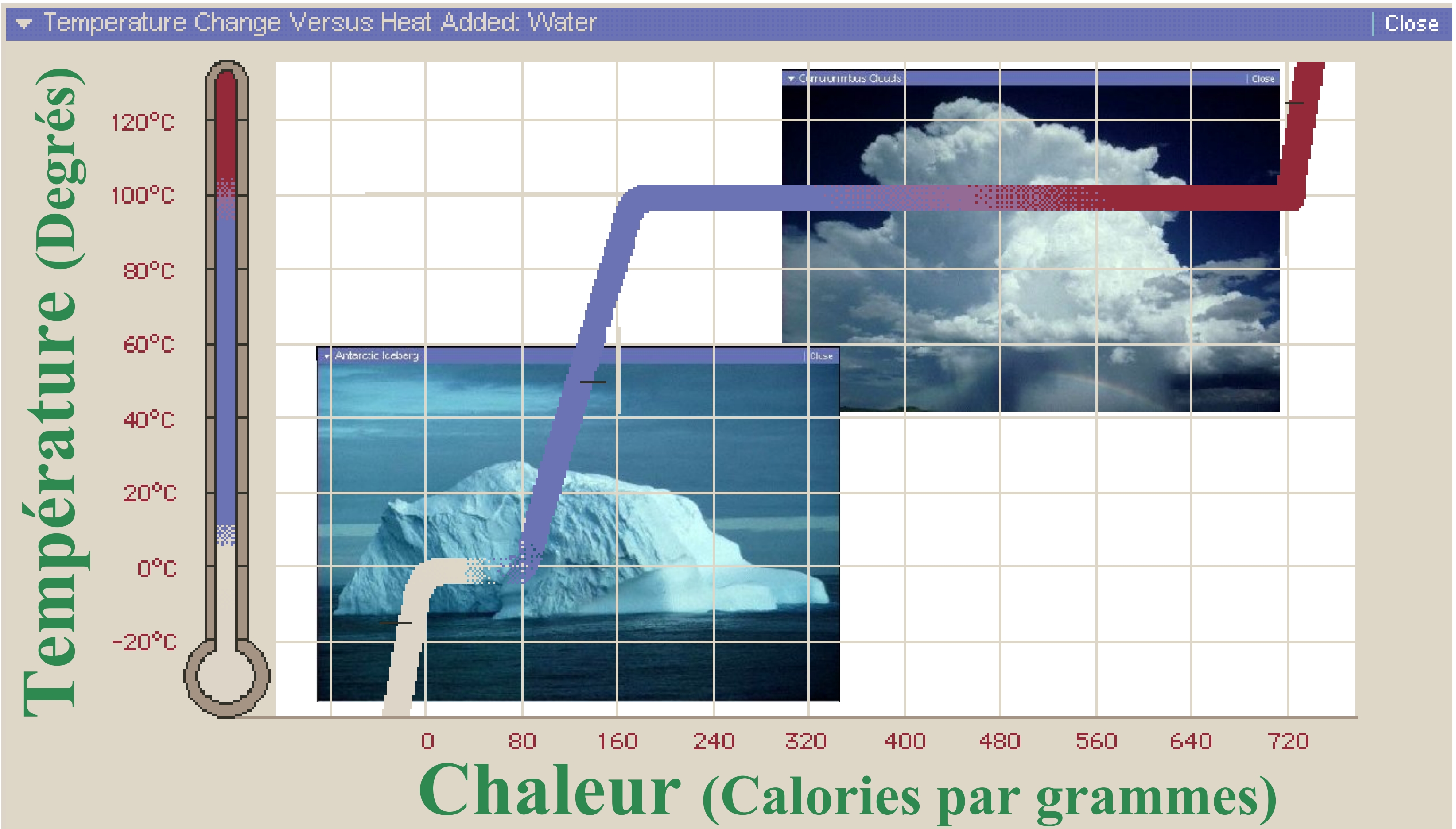
- I -

Introduction

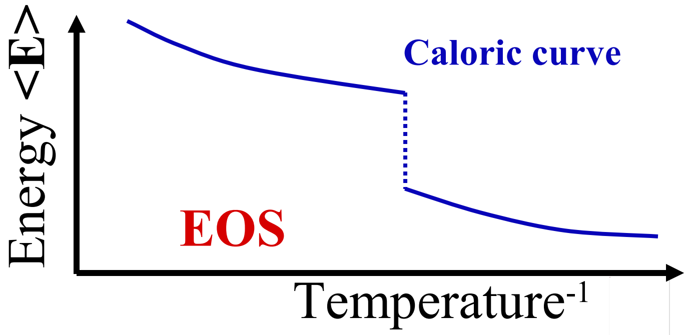
Phase transition

Anomaly in thermo

Transition de Phase



Phase transition in infinite systems



- Ex: first order:
  discontinuous EOS:

Phase transition in infinite systems

- Thermodynamical potentials: $e.g.\ F = -T \log Z$

  non analytical at $\qquad Z = \sum_{(n)} e^{-bE^{(n)}}$

  L.E. Reichl, Texas Press (1980)

- Order n:

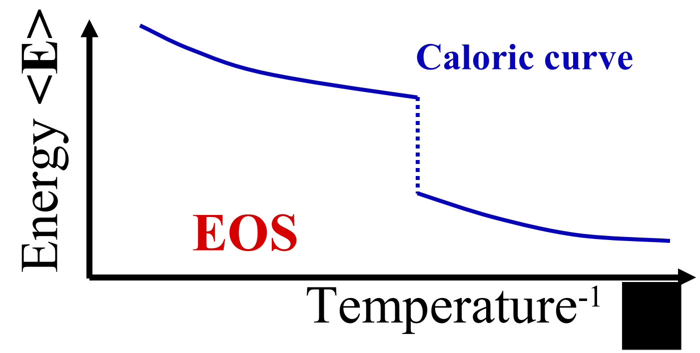
  discontinuity in

  Ehrenfest's definition

- Ex: first order:

  discontinuous EOS:

  R. Balian, Springer (1982)



Phase transitions in finite systems

- Thermodynamical potentials: e.g. $F = - T \log Z$

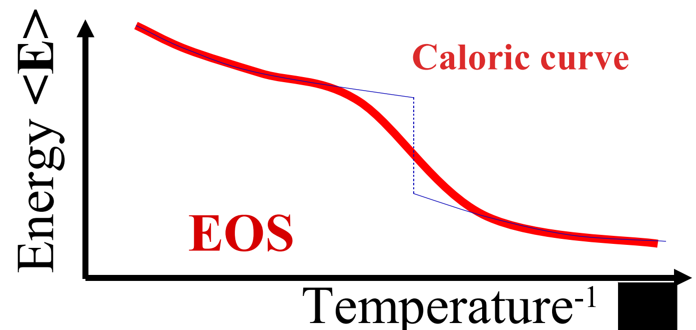
  Z analytical ε >0, [redacted]

  $Z = \sum_{(n)} e^{-\beta E^{(n)}}$

- No phase transition

  continuous

  Ehrenfest's definition



[redacted]

- Ex:

  Continuous EOS:

Phase transition in finite systems

- Thermodynamical potentials: $e.g.\ F = -T \log Z$
  Z analytical β>0, [redacted] $Z = \sum_{(n)} e^{-\beta E^{(n)}}$

- No phase transition
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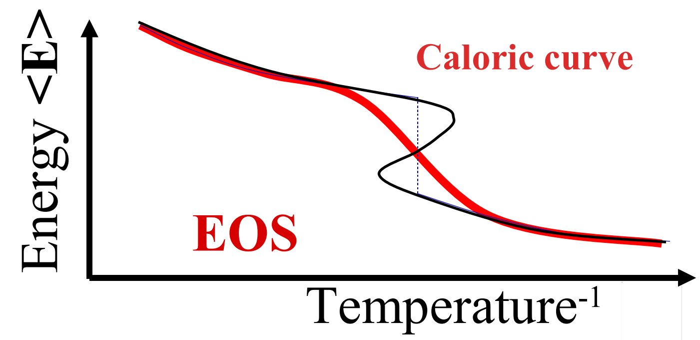
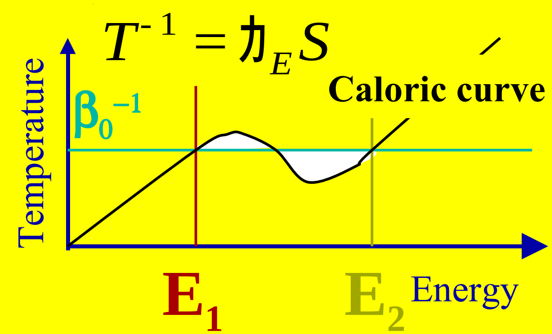
  Ehrenfest's definition



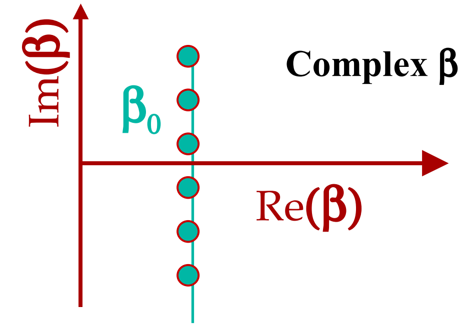
- But anomaly in the entropy
  Ex: negative heat capacity
  $C^{-1} = \partial_E T$

K. Binder, D.P. Landau Phys Rev B30 (1984) 1477;
Lynden-Bell, D. & Wood, R. 1968, Mon. Not. R. Astr. Soc. 138, 495.;
W. Thirring, Z. Phys. 235, 339 (1970),
D.H.E. Gross, Rep. Prog. Phys. 53, 605 (1990),

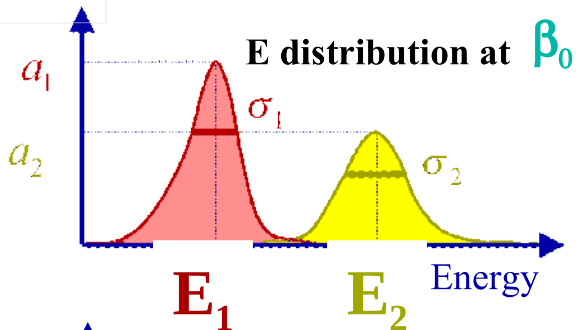


1st order in finite systems

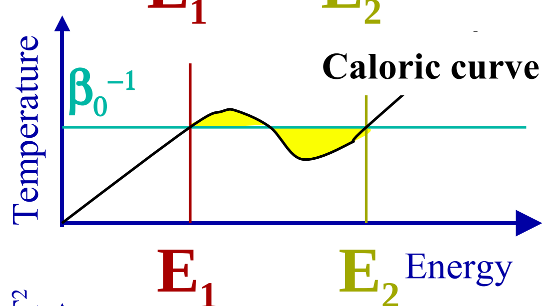
PC & Gulminelli Phys A (2003)

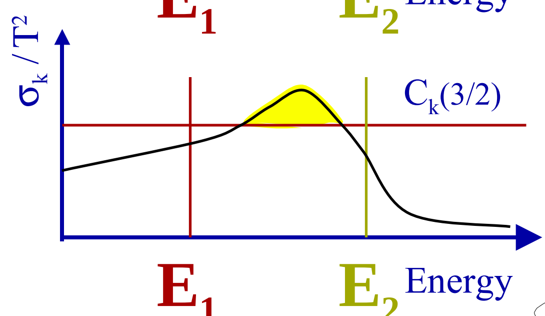


- **Free order param.** (canonical)
  - Zeroes of Z reach real axis
    Yang & Lee Phys Rev 87(1952)404
  - Bimodal distribution $(P_\beta(E))$
    K.C. Lee Phys Rev E 53 (1996) 6558



- **Fixed order para.** (microcanonical)
  - Back Bending in EOS (T(E))
    K. Binder, D.P. Landau Phys Rev B30 (1984) 1477
    Lynden-Bell, D. & Wood, R. 1968, Mon. Not. R. Astr. Soc. 138, 495.
  - Abnormal fluctuation $(\sigma_k(E))$
    J.L. Lebowitz (1967), PC & Gulminelli, NPA 647(1999)153





- II -

Origin of singularities

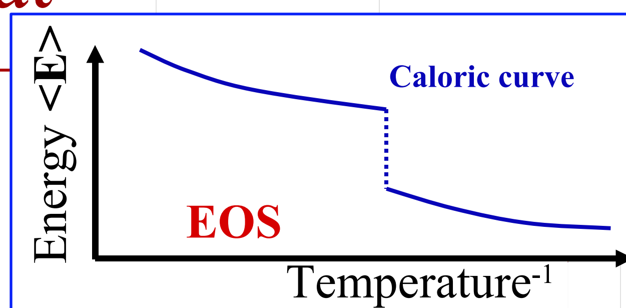
Zeroes of Z (β)

Bimodal P(E)

- Thermodynamical potentials: $e.g.\ F = -T \log Z$

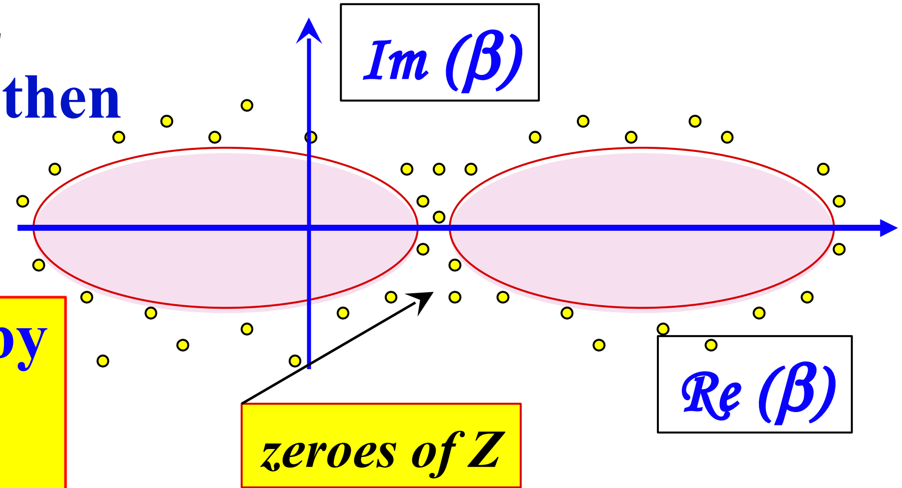
  non analytical at $Z = {}_{(n)} e^{-bE^{(n)}}$

  L.E. Reichl, Texas Press (1980)



Discontinuities from zeroes of Z

- If, when , no zeroes of Z converge on the *Real* β axis , then **logZ remains analytical**
  => No phase transition



Complex β plane

- **Phase transitions are defined by the asymptotic distribution of zeroes of the partition sum Z**
  (C.N.Yang T.D.Lee 1952)

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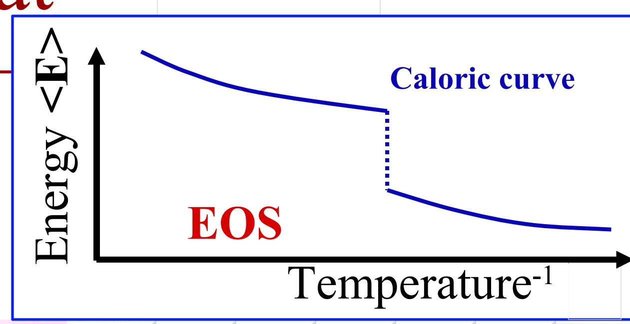

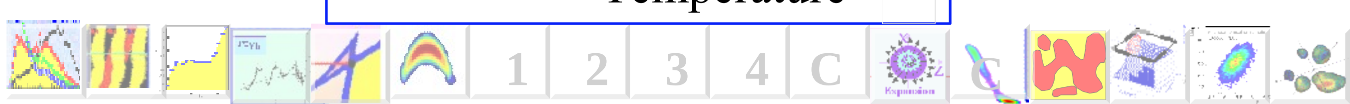

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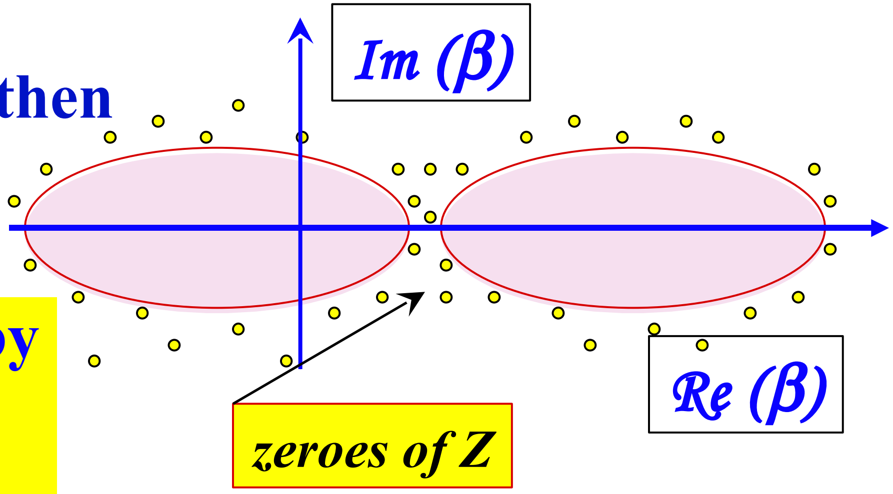

Complex β plane

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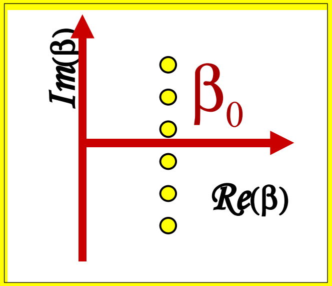
- First order phase transitions
  A uniform density of zero's on a line crossing the real β axis perpendicularly at β_0

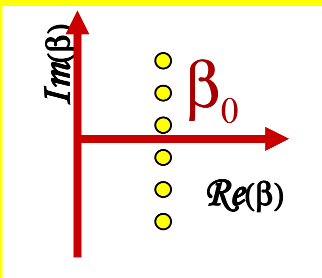


Yang - Lee theorem

First order phase transitions

A uniform density of zero's on a line crossing the real β axis perpendicularly at β_0

Yang - Lee theorem



A theoretical definition

- Only valid asymptotically: $V = \infty$
- How to extend it, $V \neq \infty$?

How to use it experimentally ?

- **First order phase transitions**
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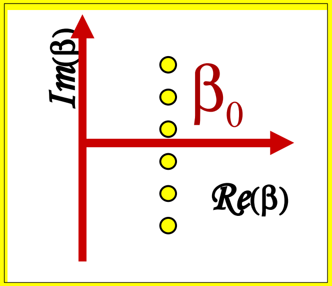

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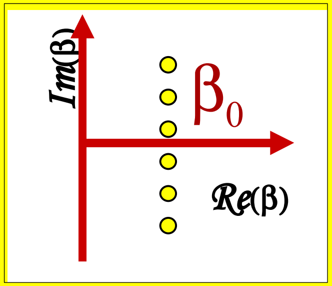

Link with energy distribution P(E)

- Z Laplace transform of P:

Ph. Ch., F. Gulminelli, Physica A 2003

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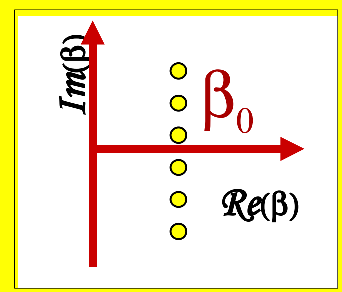
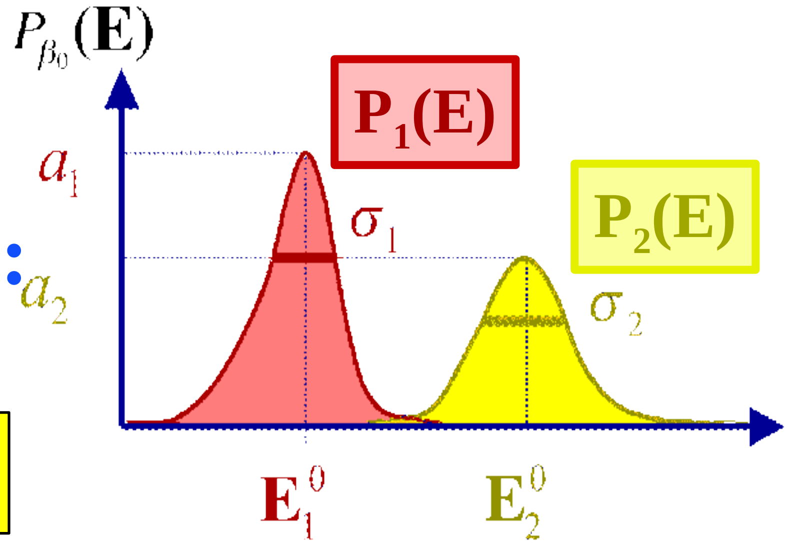


Link with energy distribution P(E)

- Z Laplace transform of P:
- Bimodal P with $\Delta E \propto N$



Ph. Ch., F. Gulminelli, Physica A 2003

$\Leftrightarrow$

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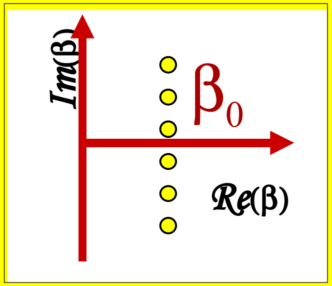
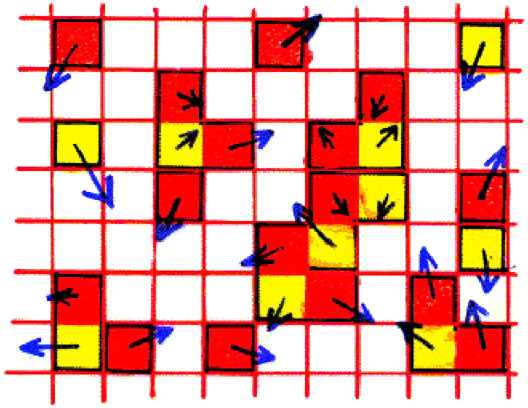
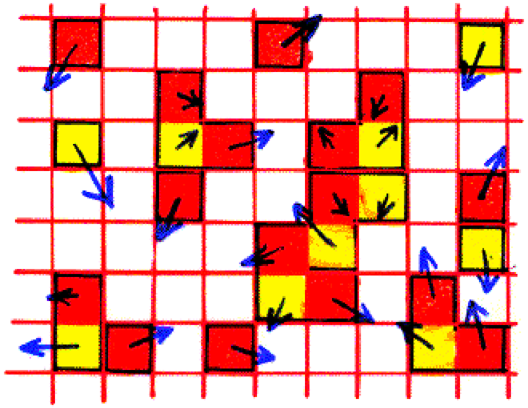


Lattice gas model

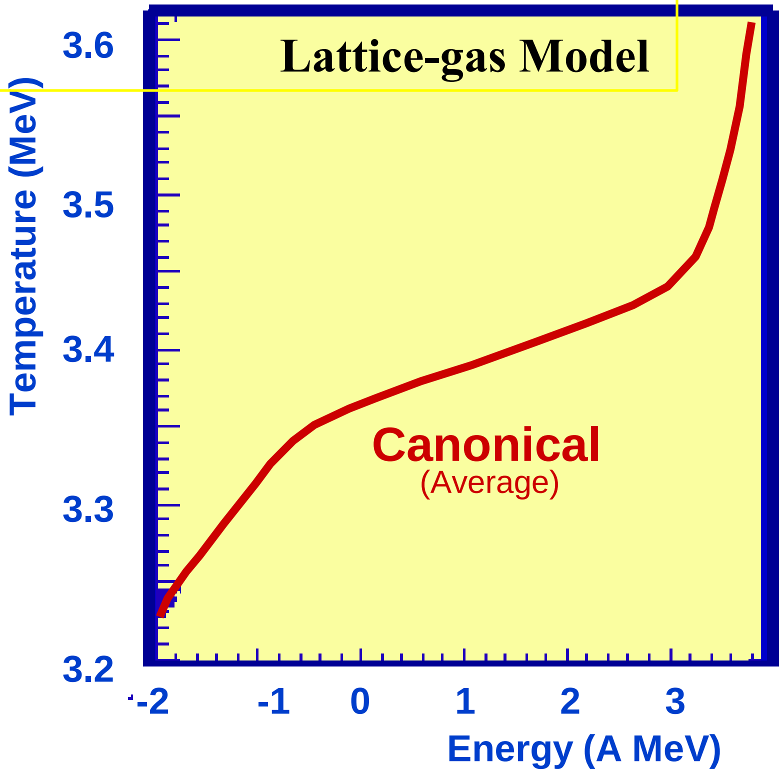


- **Closest neighbors interaction**
- **Metropolis Boltzmann weight $e^{-\beta E}$**
- **Average volume <V> (pressure)**

Lattice gas model



- <E> smooth fonction of β
  (canonical caloric curve)



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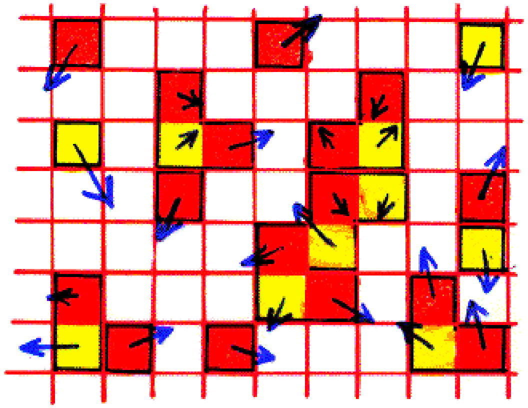

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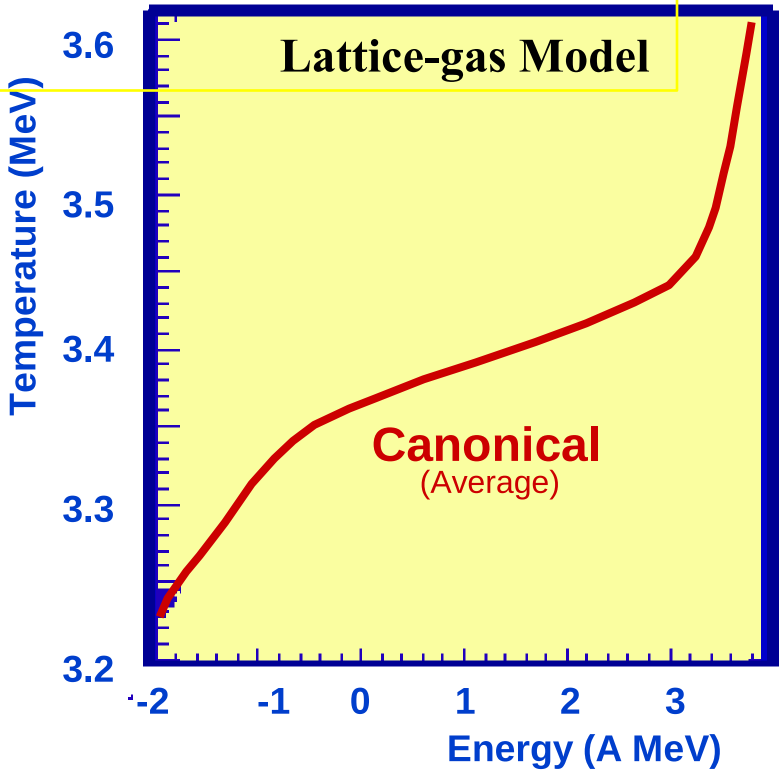

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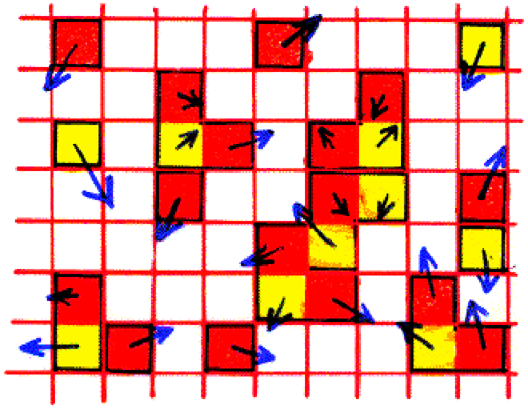
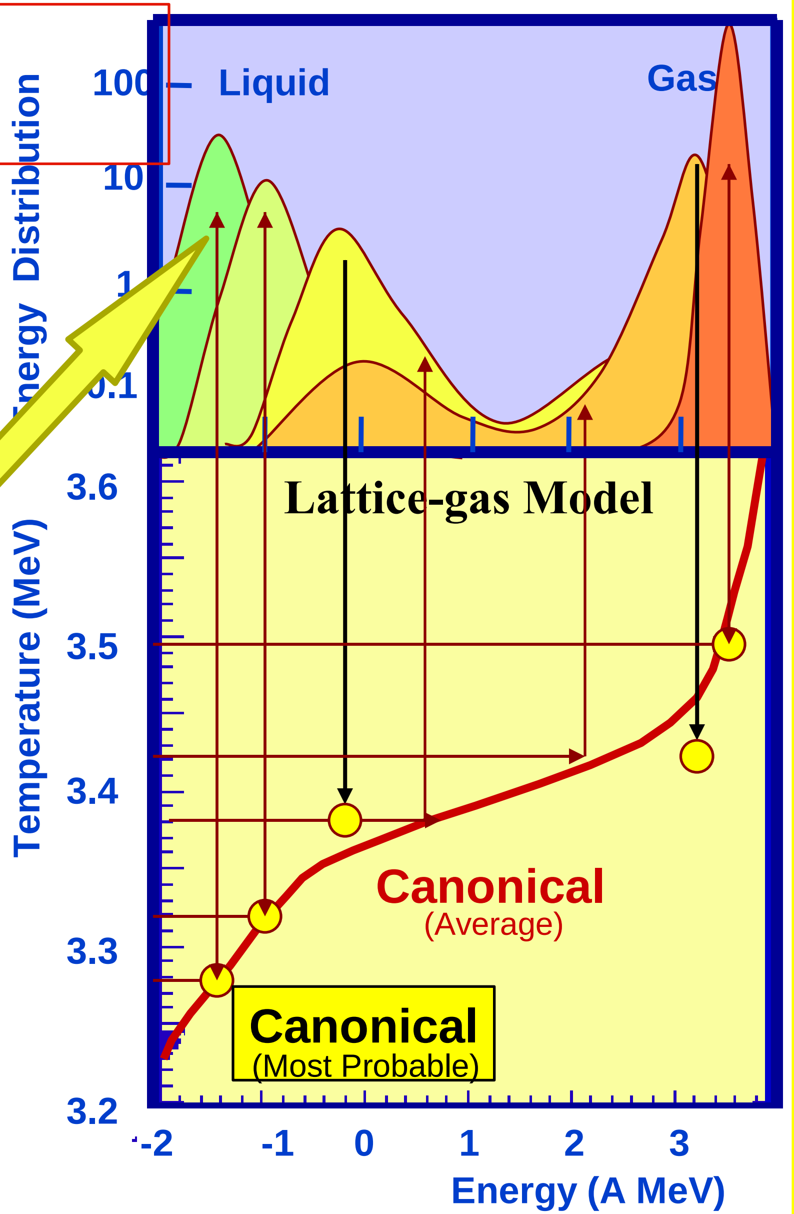
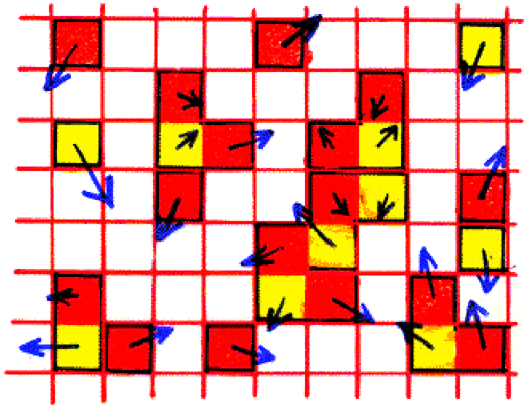


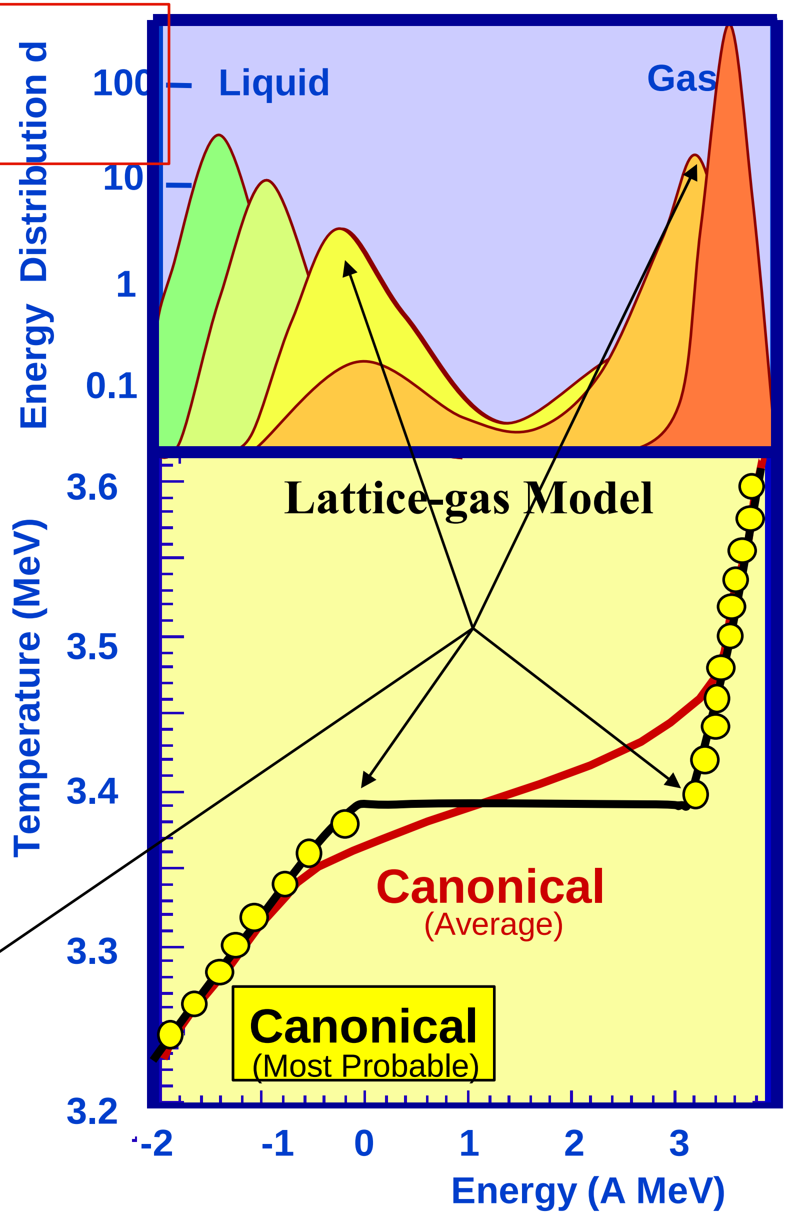
- Energy distribution at a fix temperature



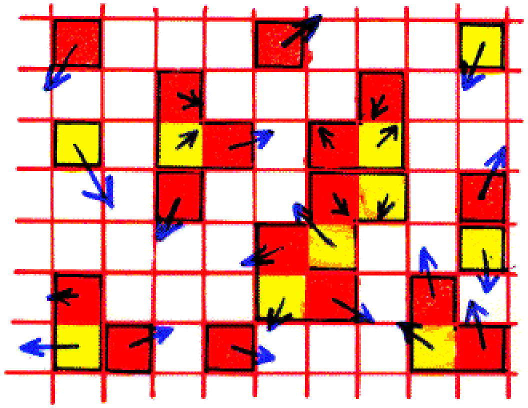
Lattice-gas model



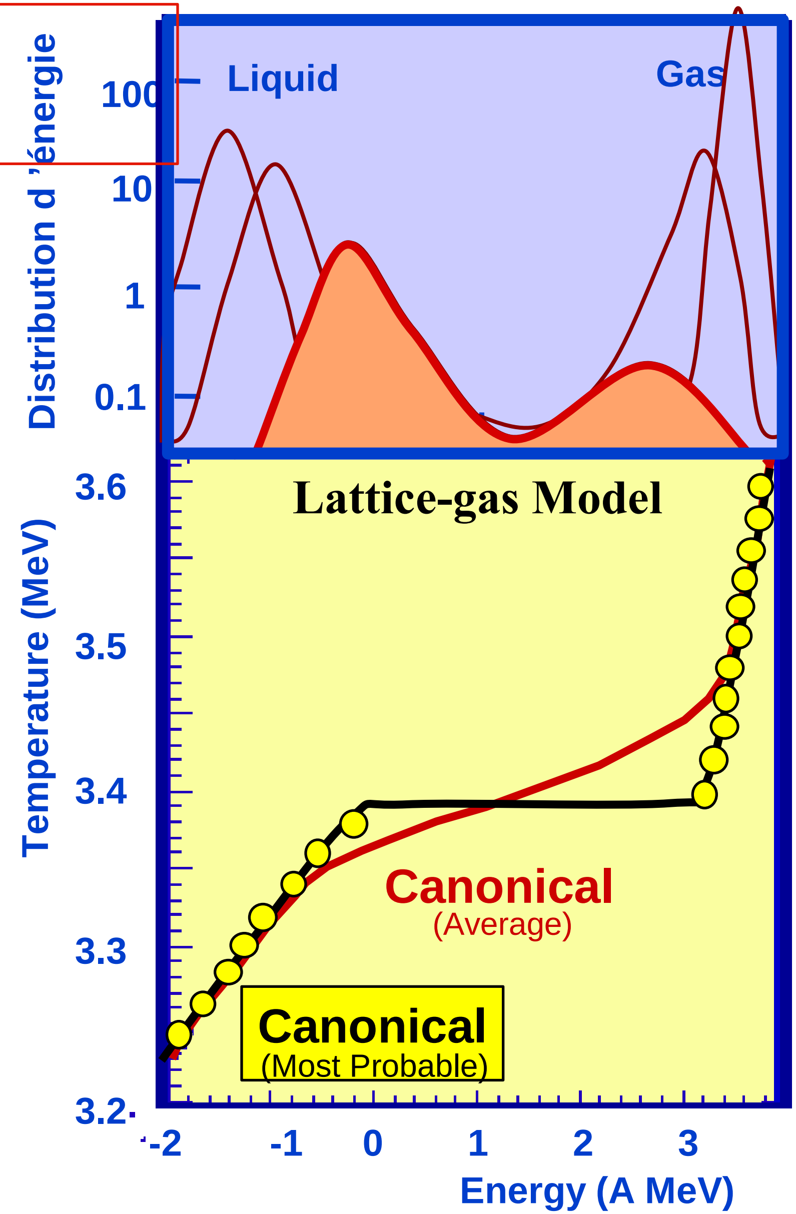
- Energy distribution at a fixed temperature
- Discontinuity of the most probable

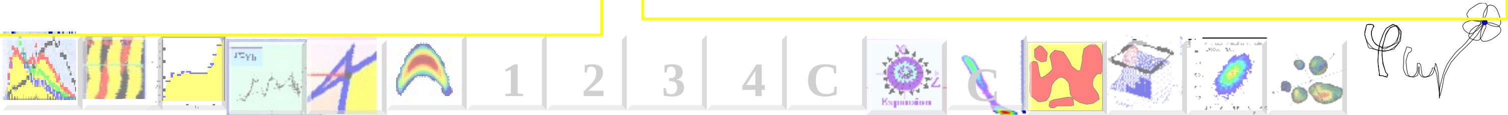


Lattice-gas model



- Energy distribution at a fixed temperature
- Bimodal distribution
- Discontinuity of the mos probable





A first order phase transition in finite systems

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A bimodal (event) distribution of energy

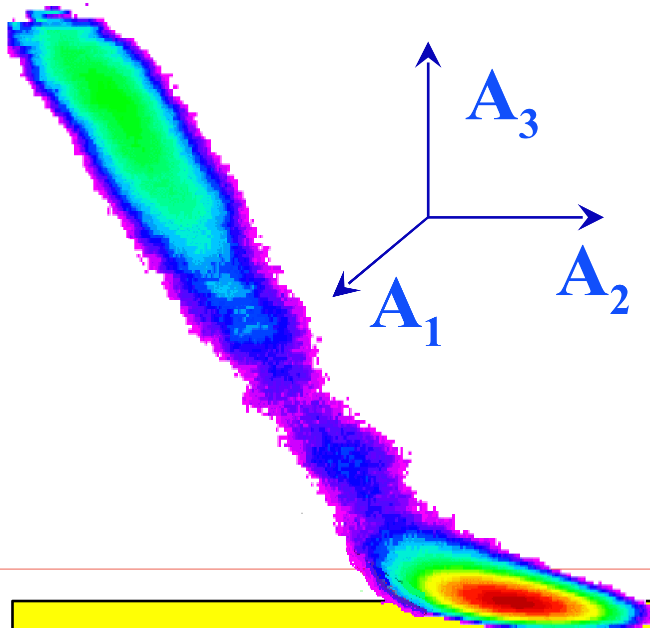
If the energy is free to fluctuate (canonical - energy reservoire)

A first order phase transition in finite systems

⇕

A bimodal (event) distribution of an observable (order parameter)

If this observable is free to fluctuate (intensive ensemble)



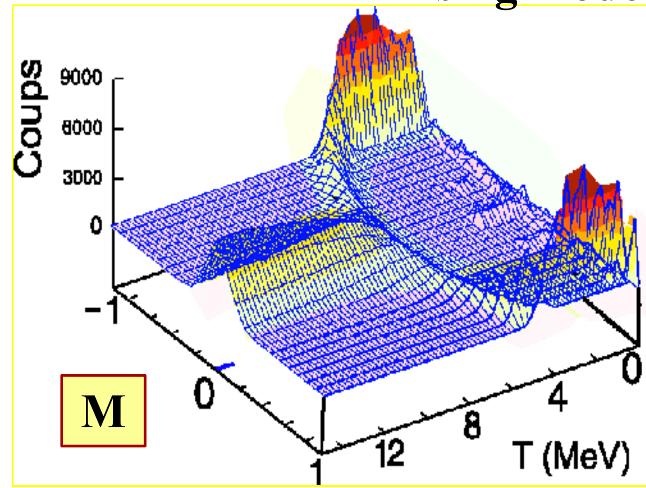
- **Access to order parameters**
  - **Nature of order parameter:**
    - **Is it collective?**
  - **Scaling of the jump with N:**
    - **What happen at the thermo limit ($N = \infty$) ?**
    - **Is it a macro. phase transition?**

A bimodal (event) distribution of an observable (order parameter)

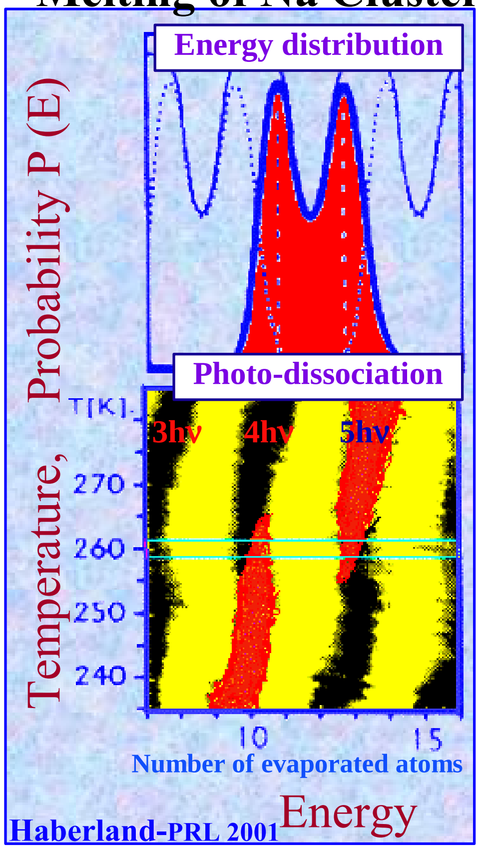
If this observable is free to fluctuate (intensive ensemble)

Bimodal distributions

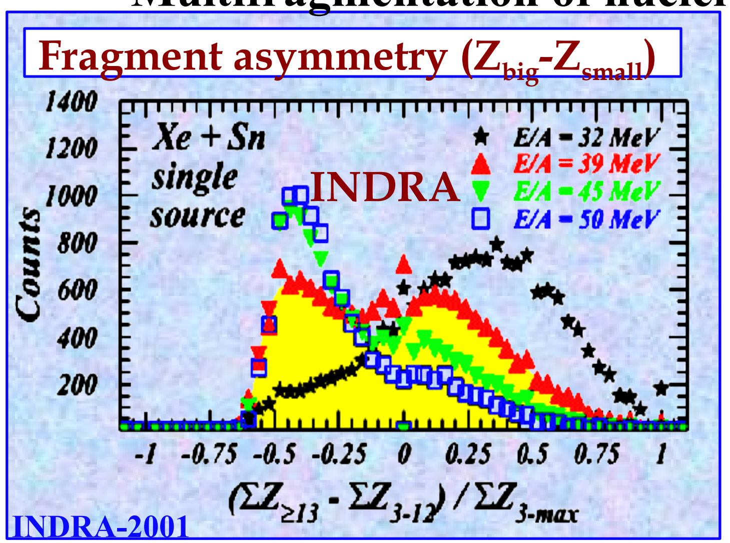
Ising model



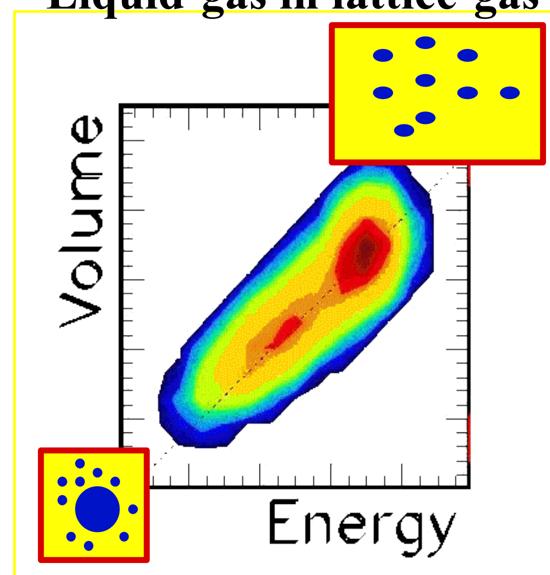
Melting of Na Cluster



Multifragmentation of nuclei



Liquid-gas in lattice-gas



Melting of Na_{147}^{+}

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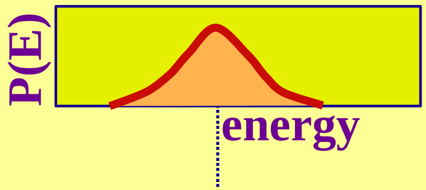

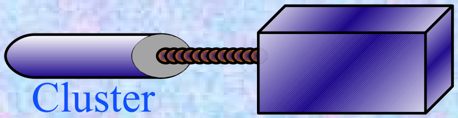

M.Schmidt et al, Nature 1999, PRL 2001

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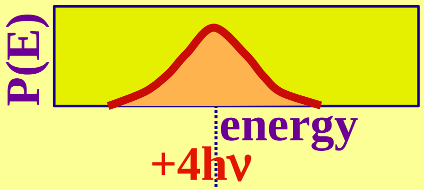

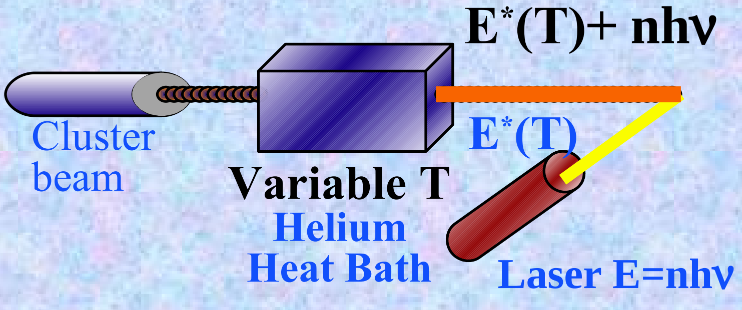

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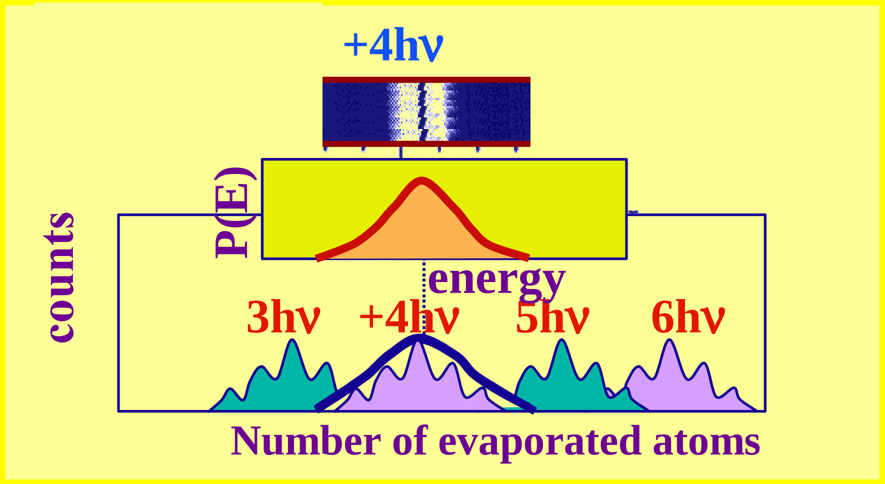

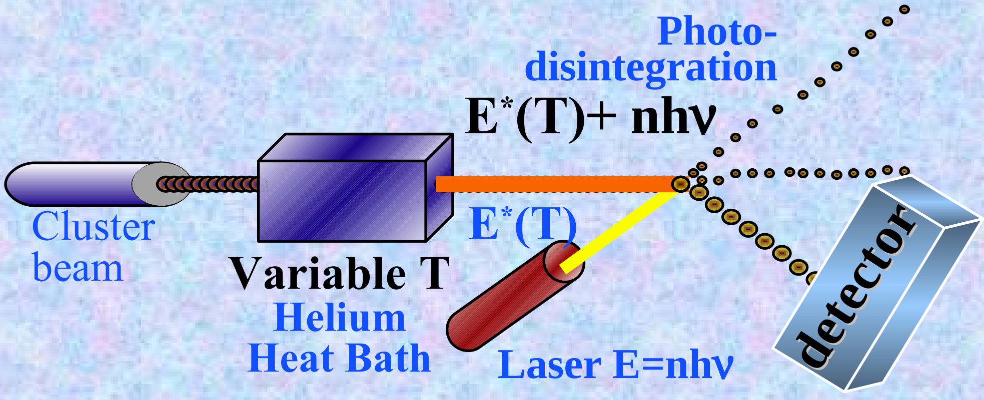

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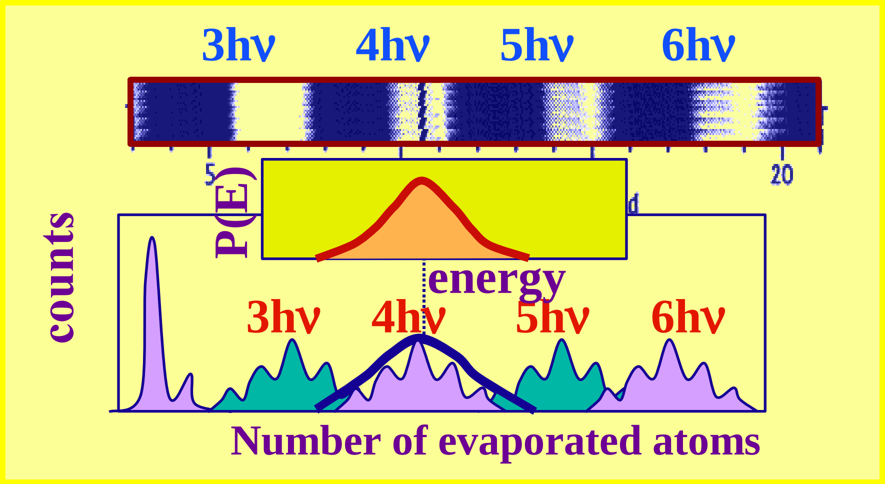

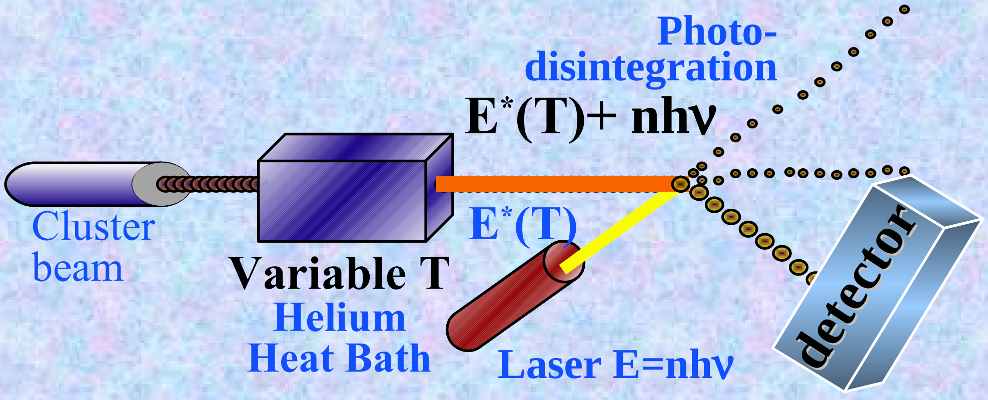

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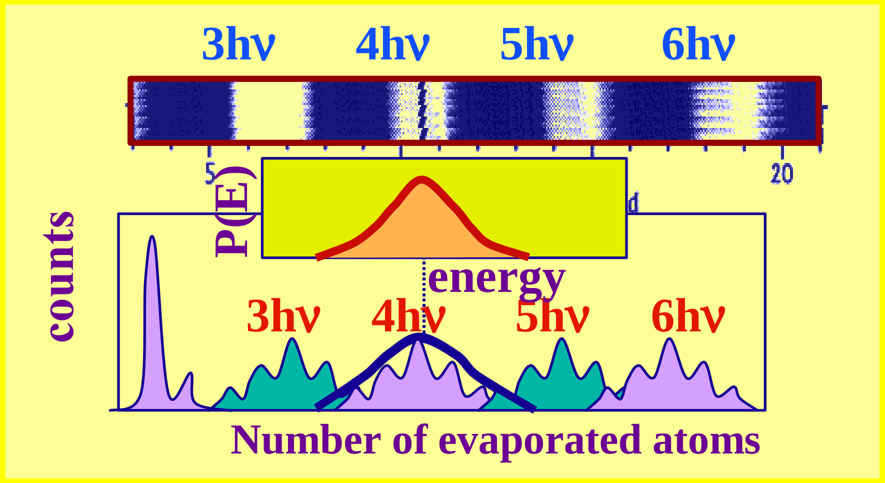

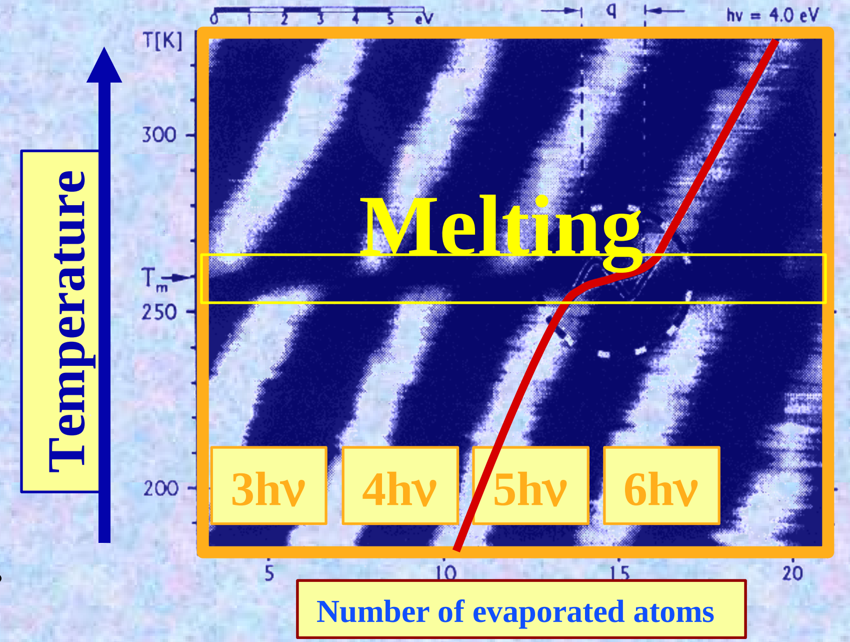

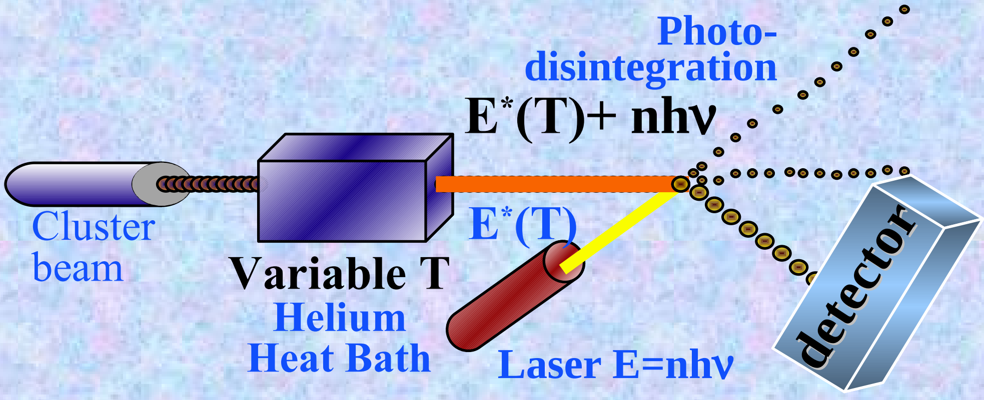

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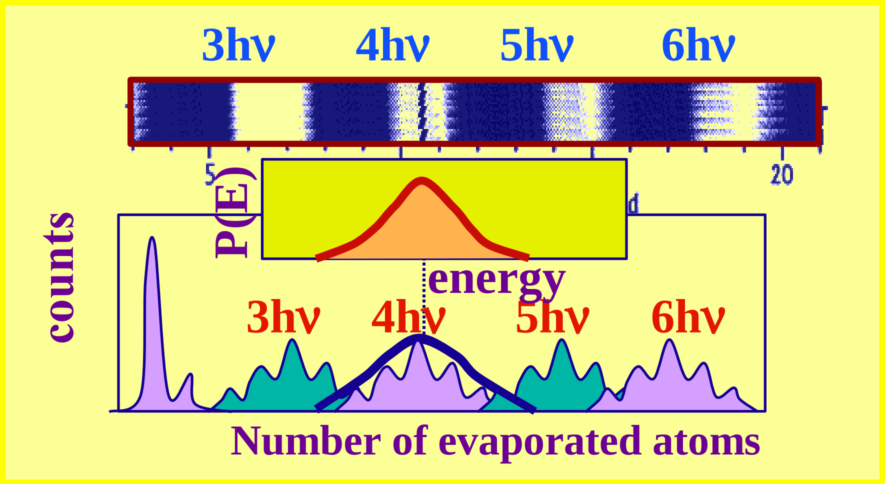

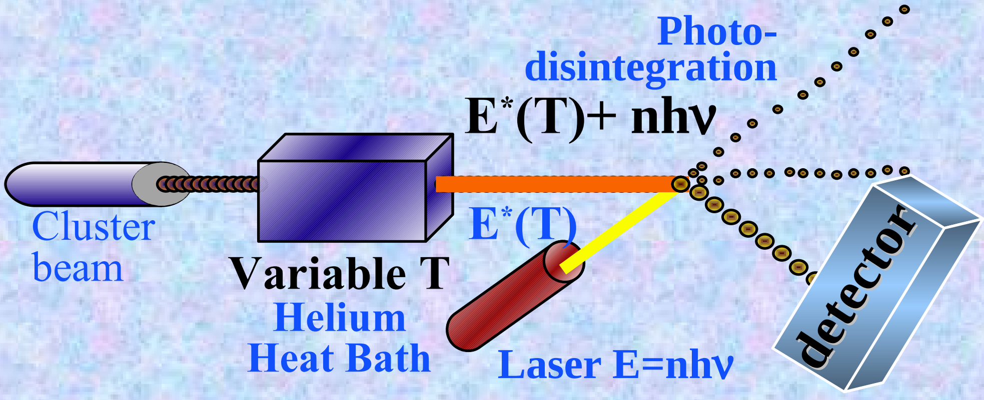

Number of evaporated atoms

M.Schmidt et al, Nature 1999, PRL 2001

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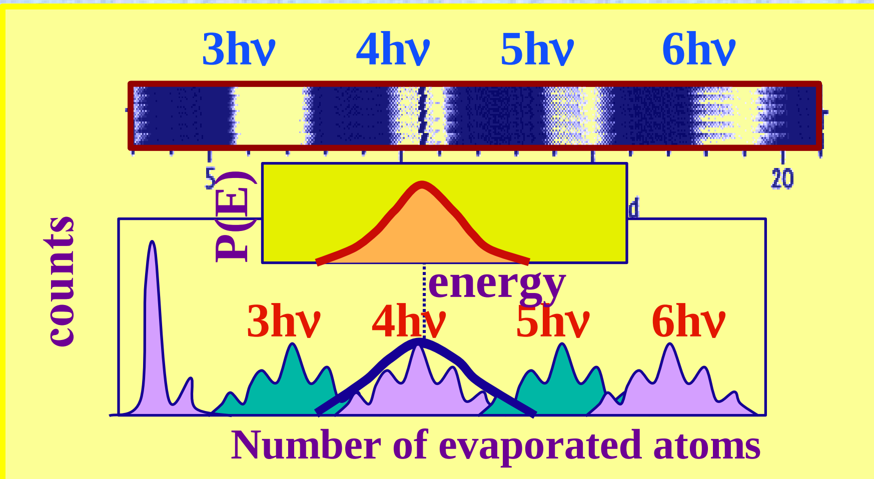

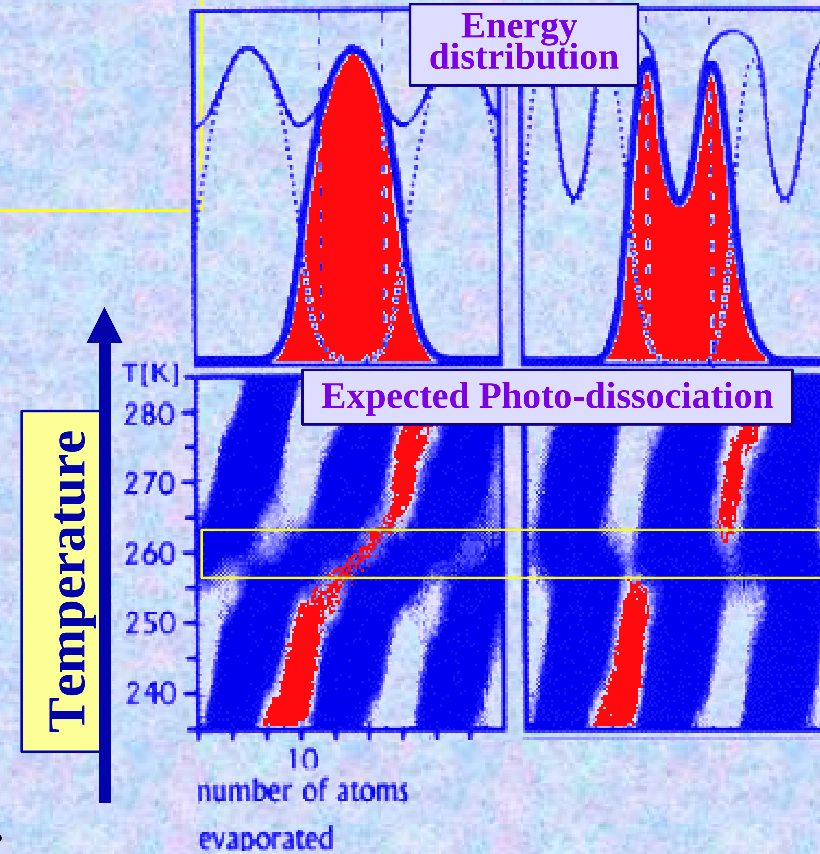

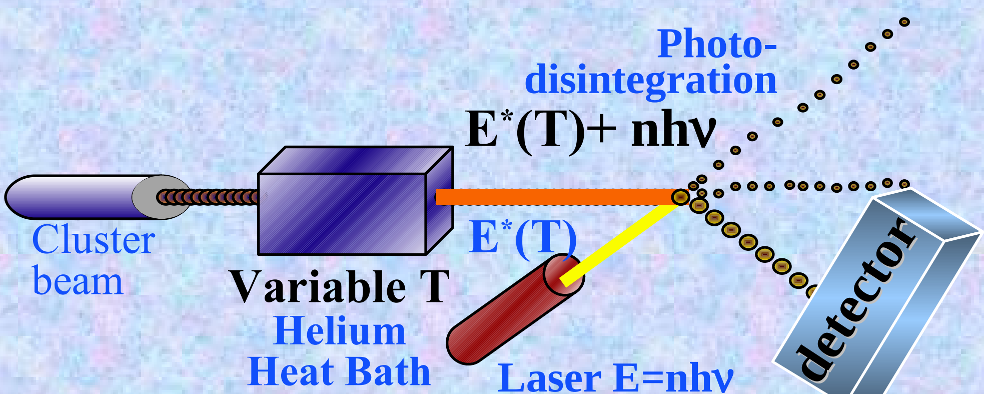

Na_{147}^{+}

(hν = 3.1 eV) :

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M.Schmidt et al, Nature 1999, PRL 2001

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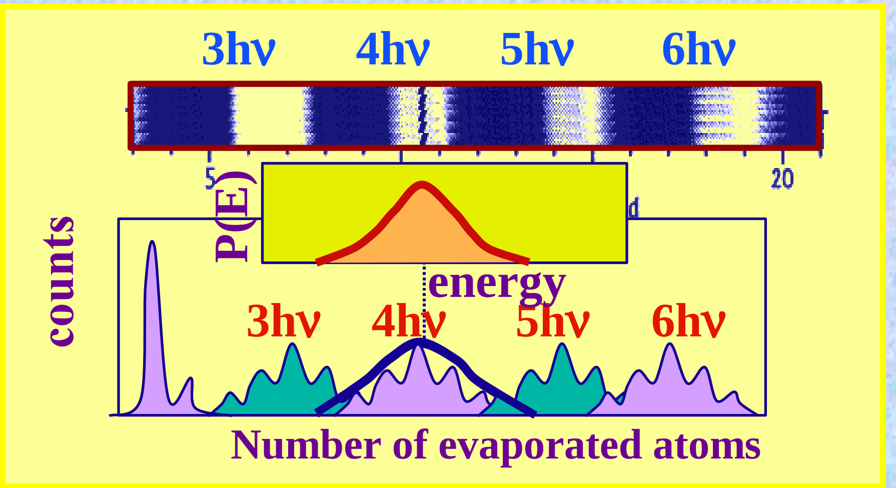

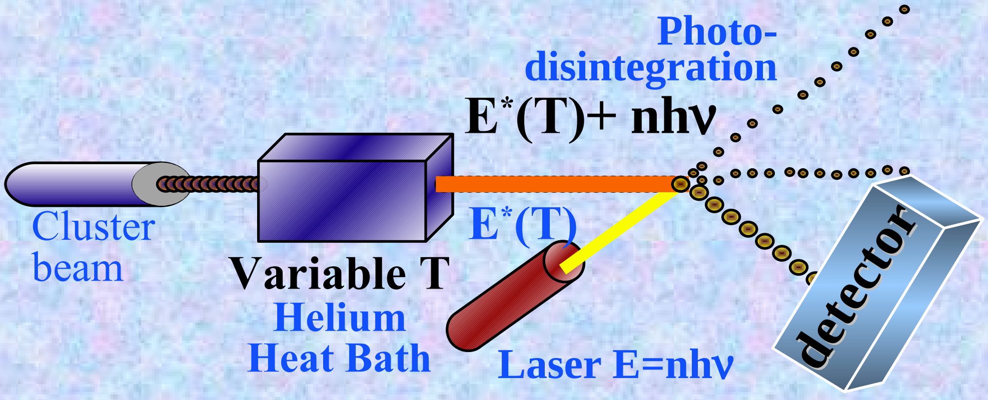

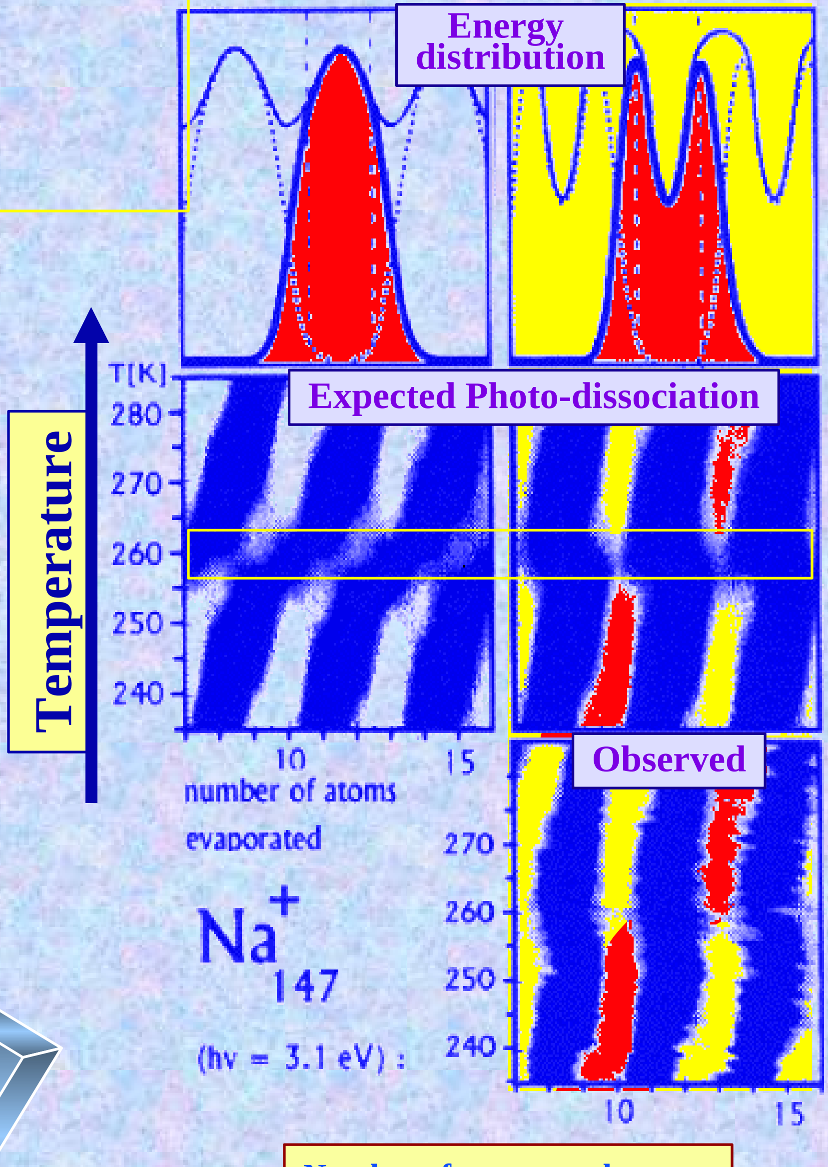

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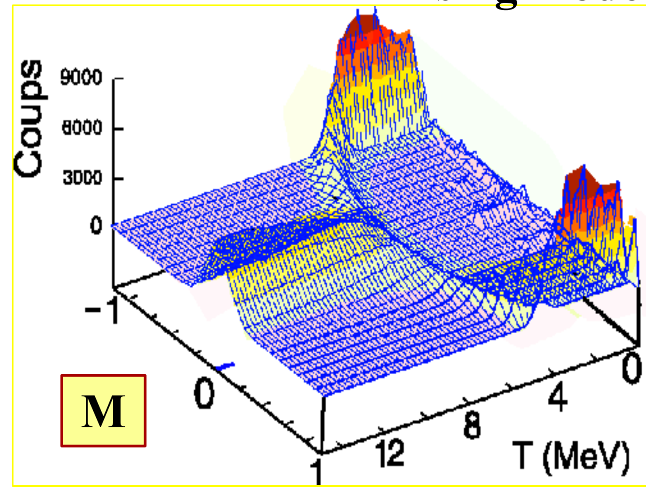
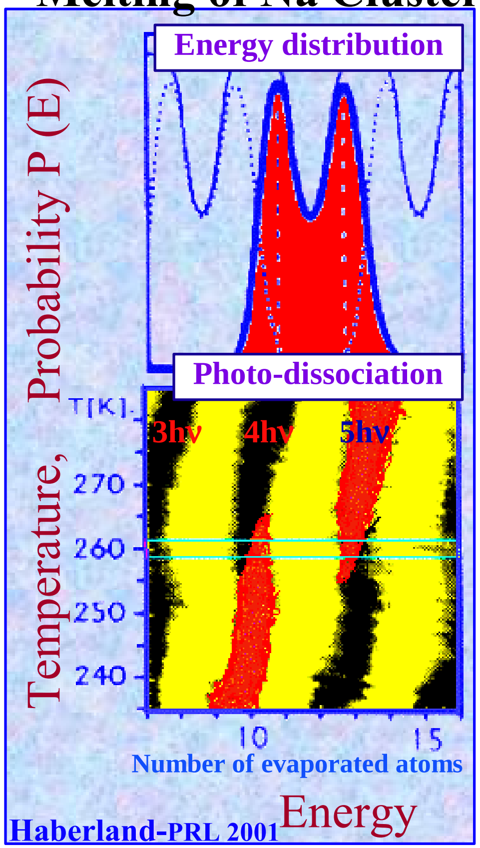
Bimodal distributions

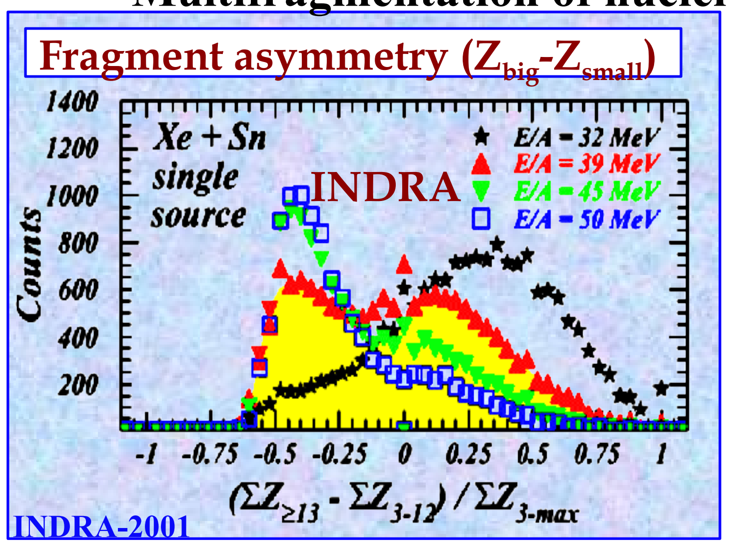
Ising model



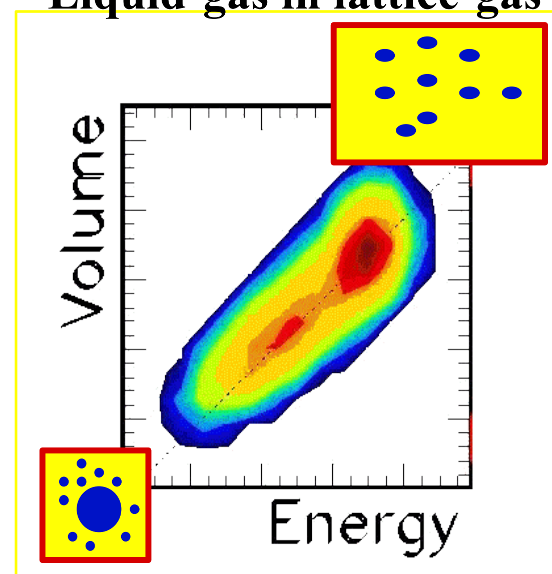
Melting of Na Cluster



Multifragmentation of nuclei



Liquid-gas in lattice-gas



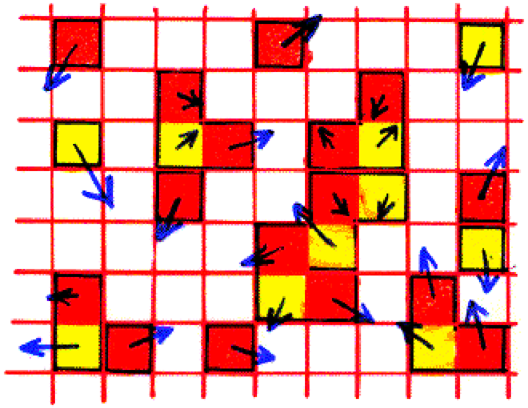
- III -

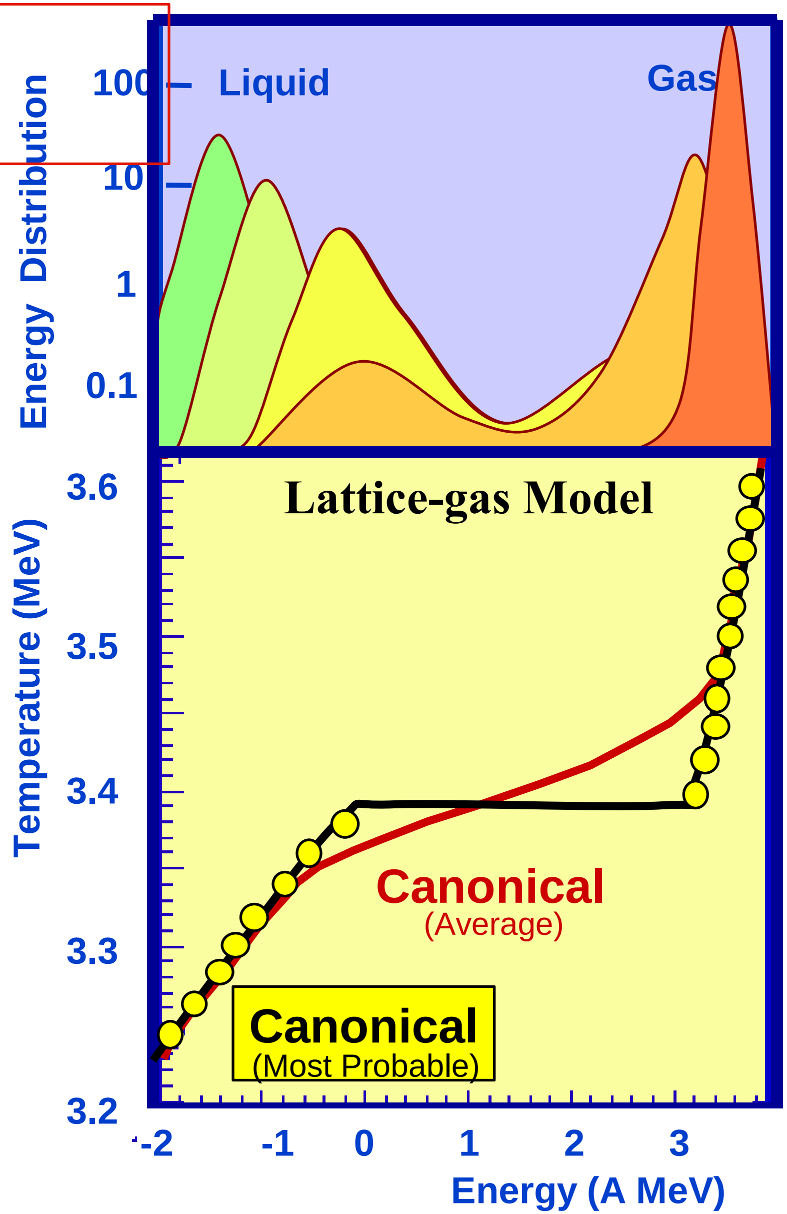
Convex S

Back bending T

Negative heat capacity

Lattice-gas model

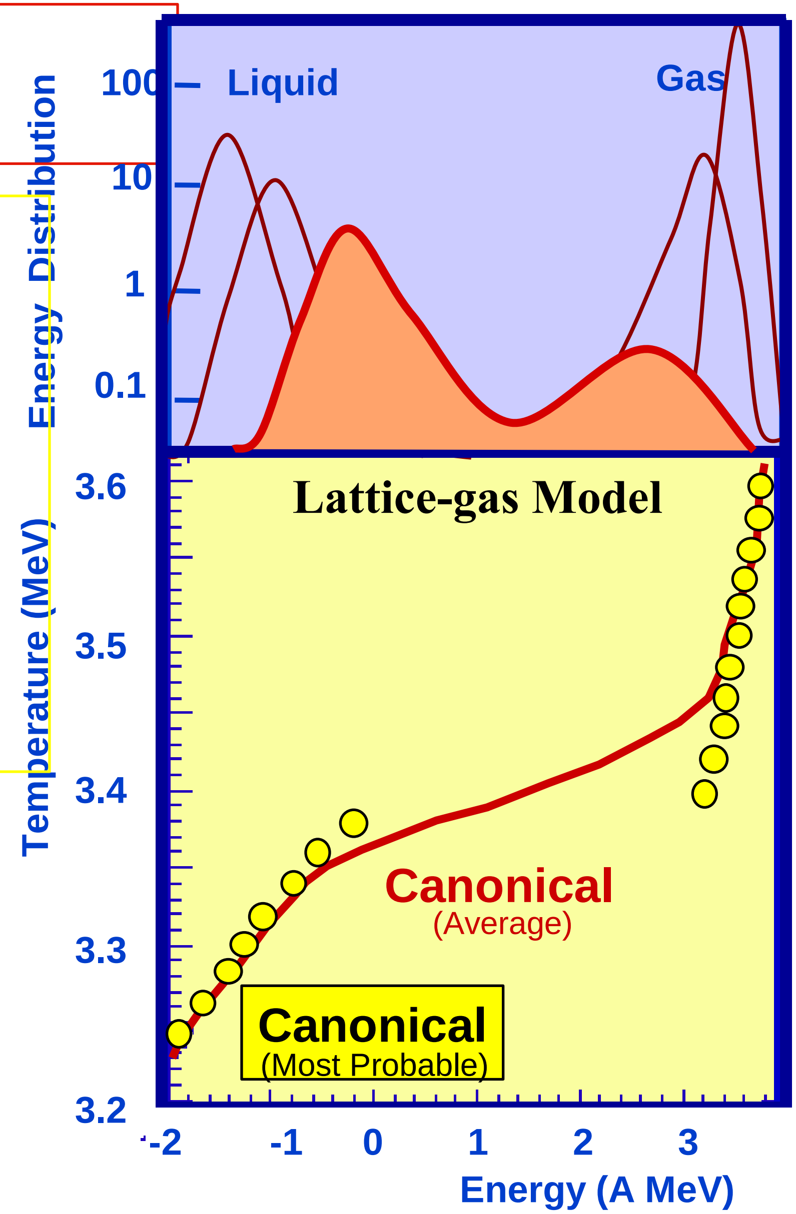




- Energy distribution at a fixed temperature
- Discontinuity of the mos probable

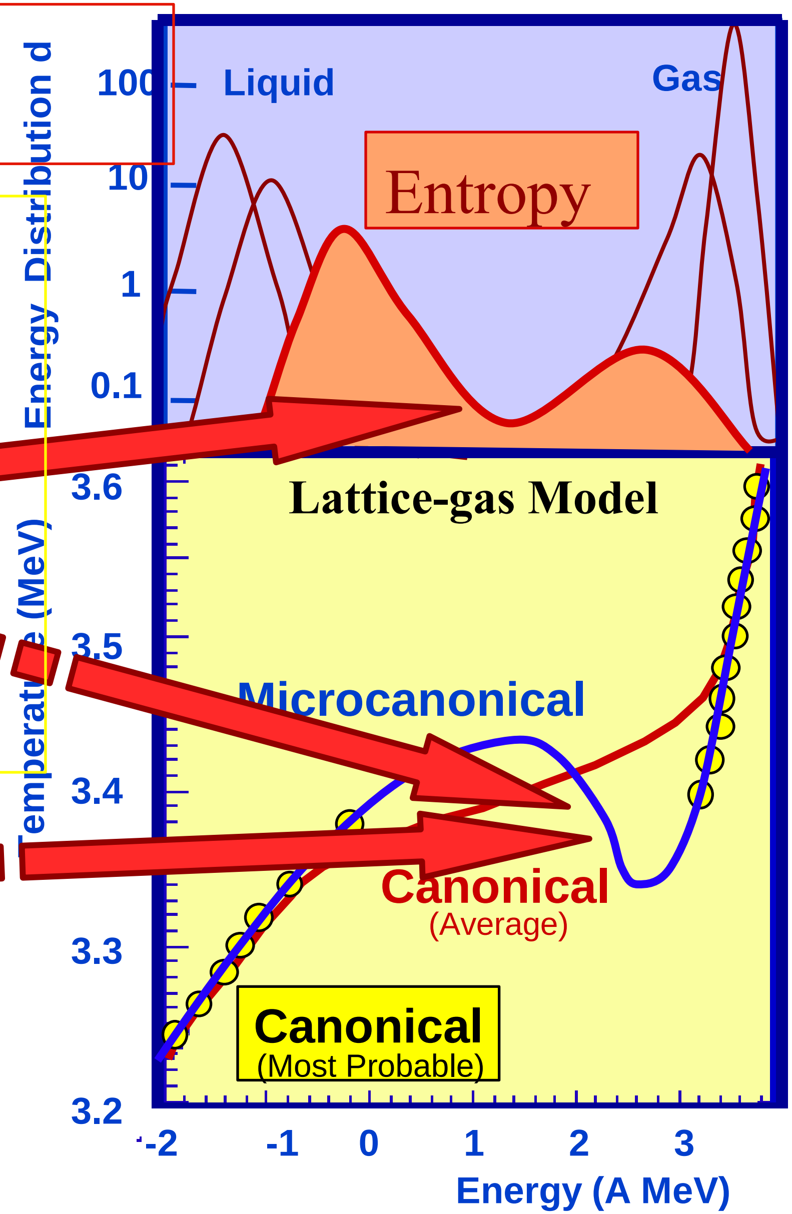
Lattice-gas model

- E distribution
  - ✦ $P_\beta(E) = W(E) e^{-\beta E} / Z_\beta$



Lattice-gas model

- Microcanonic: S=logW
  - $\log Z_\beta P_\beta(E) = S(E) - \beta E$
- **Convex Entropy**
- Decreasing T: $T^{-1} = d_E S$
  - $T^{-1} = \beta - d_E(\log P_\beta)$
- Negative Heat Capacity



A first order phase transition in finite systems

⇕

A negative heat capacity

If the energy is fixed (microcanonical)

A first order phase transition in finite systems

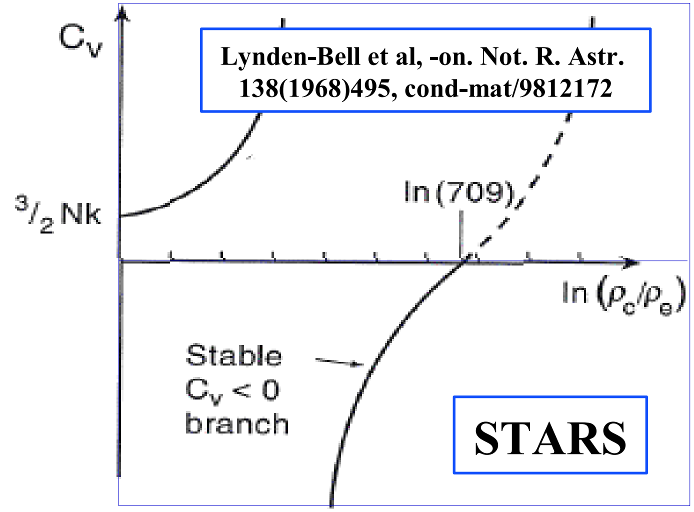
⇕

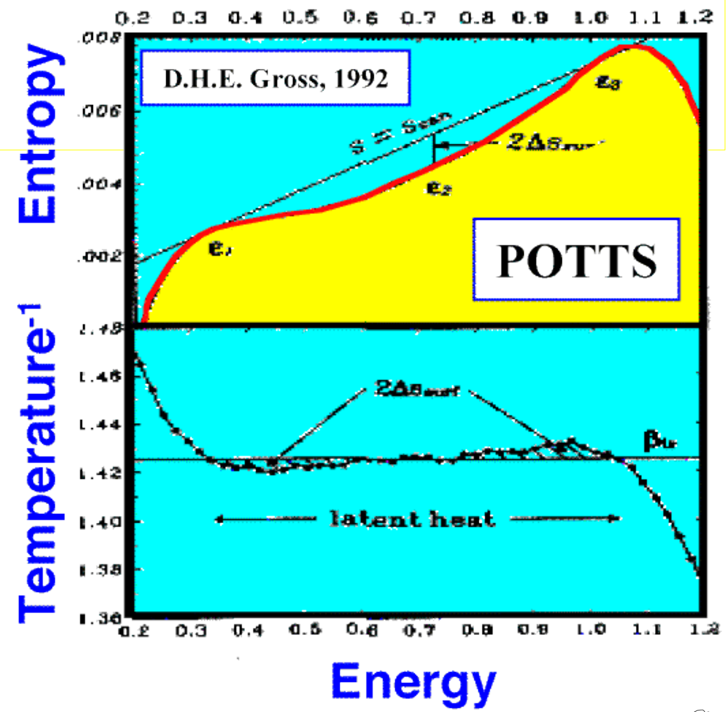
A inverted curvature of the thermodynamical potential

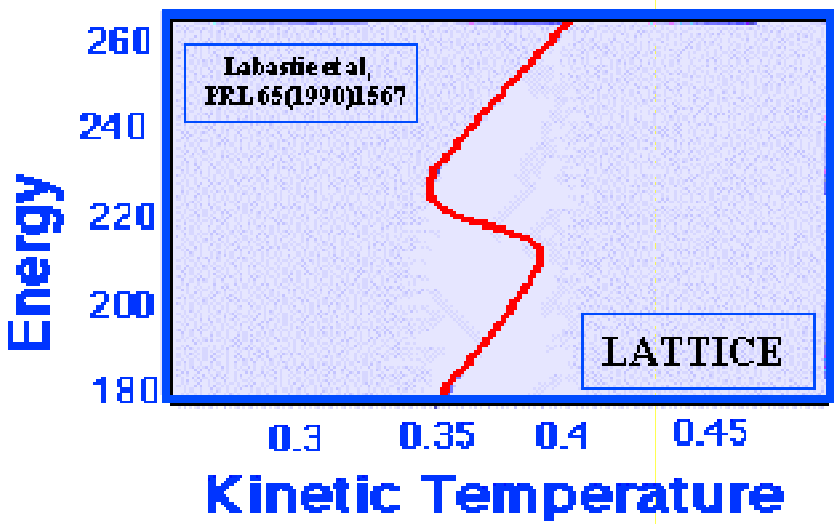
If the observables (order param.) are fixed (intensive ensemble)

Negative Heat Capacity

First Order Phase Transition in Finite Systems

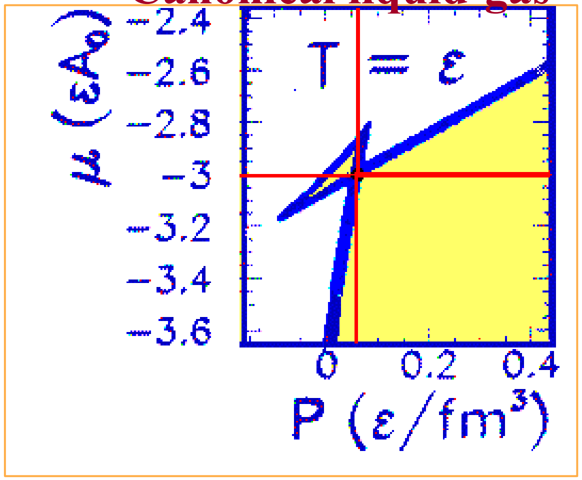






Abnormal curvatures

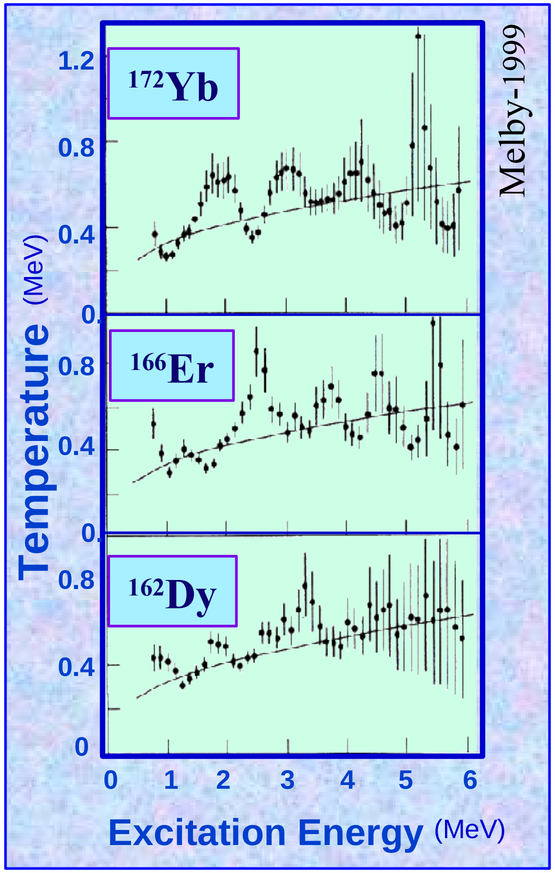
Canonical liquid-gas



F. Gulminelli, Ph. Ch. PRL(1998)

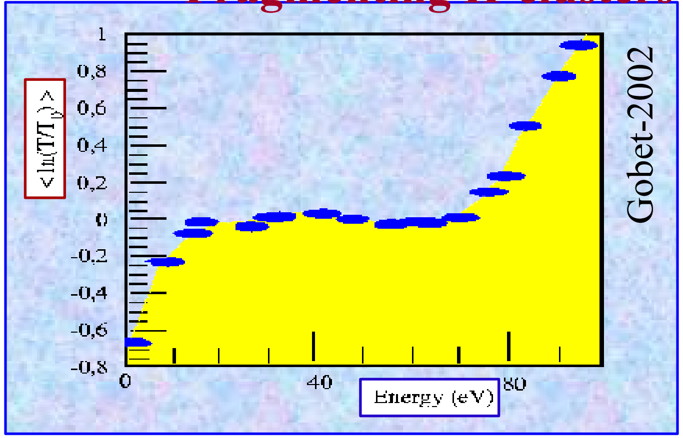
Compressibility < 0
Susceptibility < 0

Nuclear superfluidity



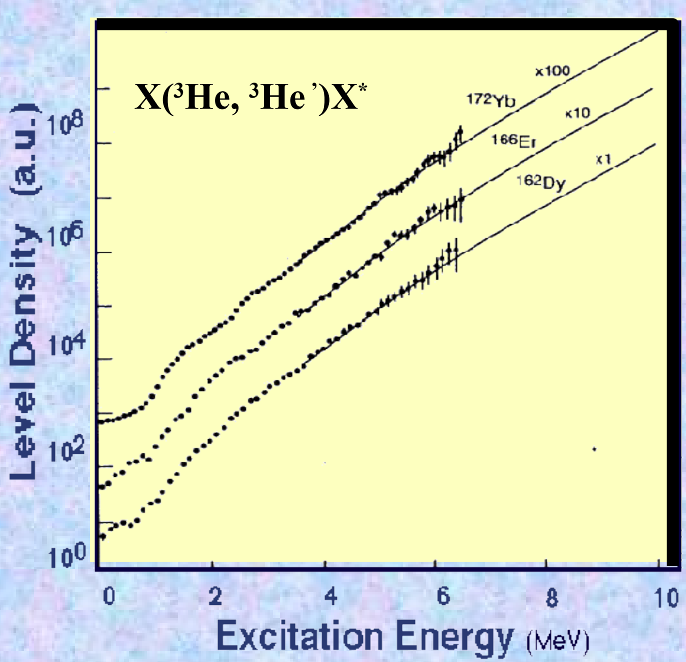
Convex entropy

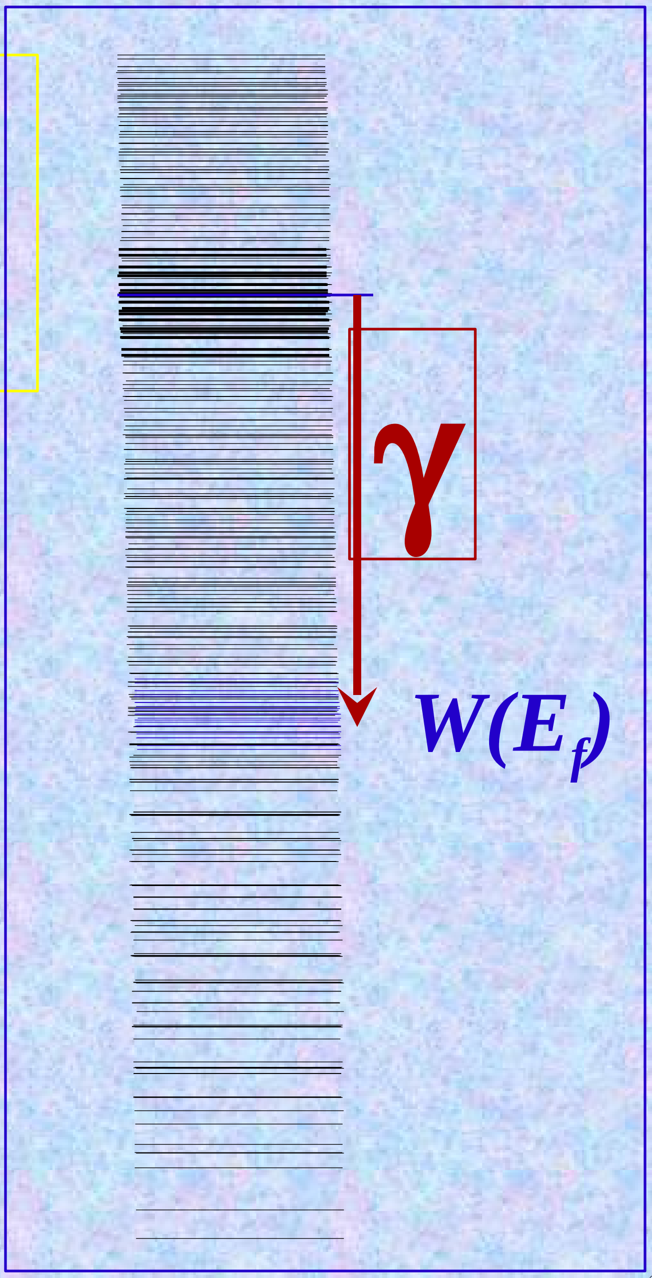
Fragmenting H-clusters



T decreasing with E

At low energy
γ Decay: level density



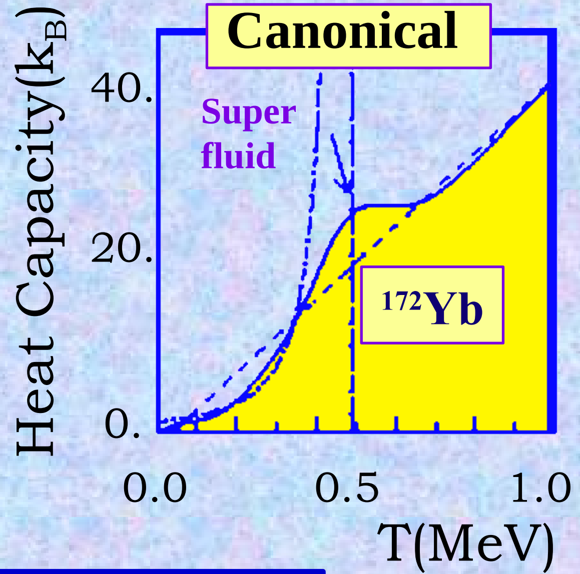


Melby et al, PRL 83(1999)3150

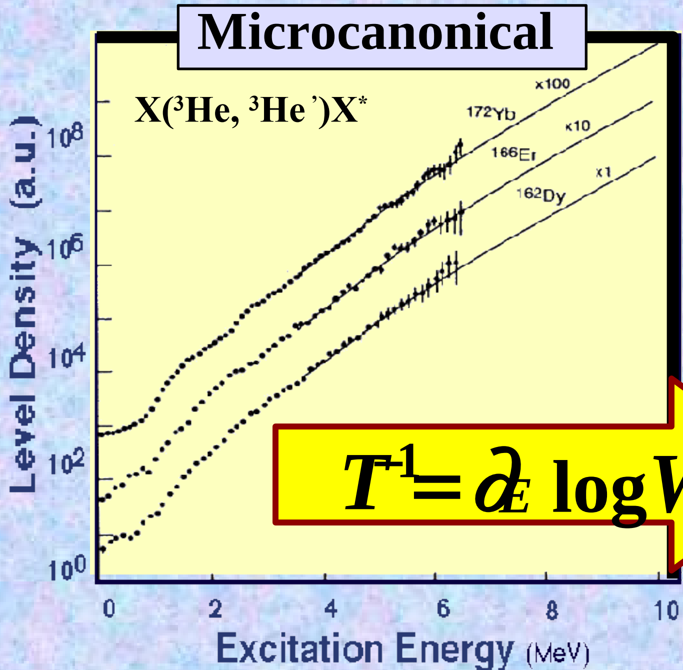
Superfluid Nuclei

γ Decay: level density

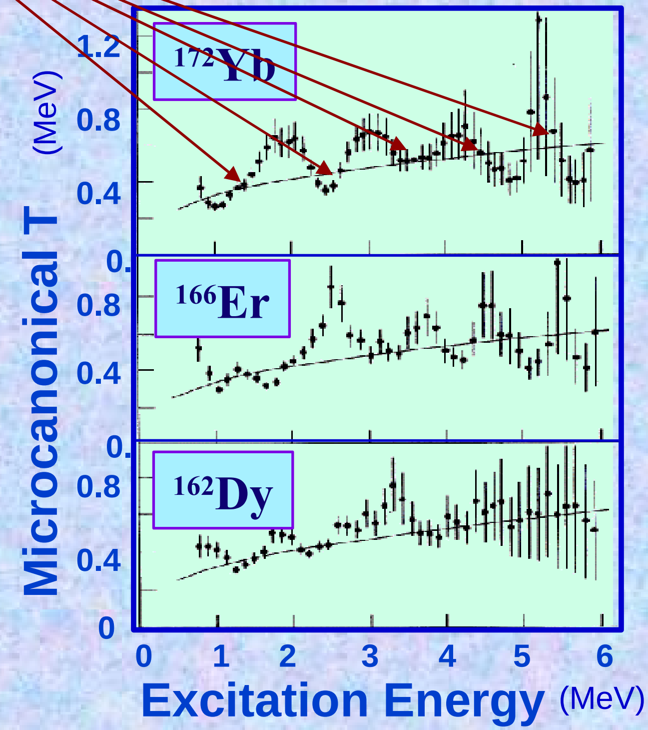
- **Breaking of pairs**



W = exp S



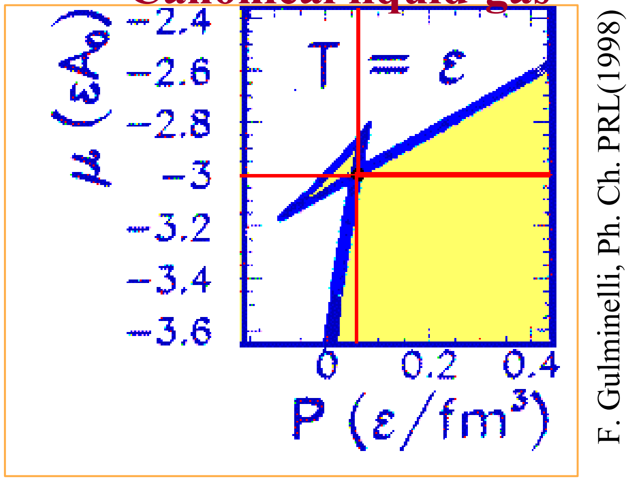
$$T^{-1} = \partial_E \log W$$



Melby et al, PRL 83(1999)3150 A.Schiller et al. 2000

Abnormal curvatures

Canonical liquid-gas



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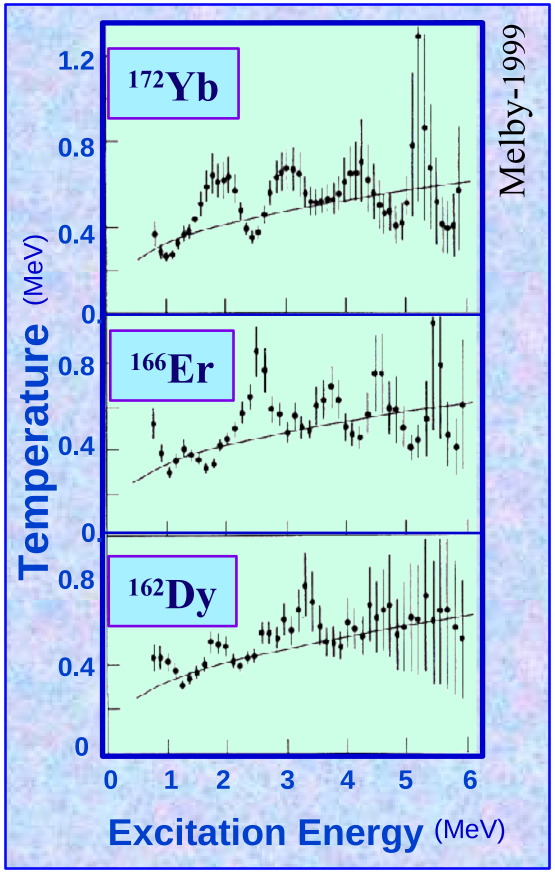
Nuclear superfluidity



Convex entropy

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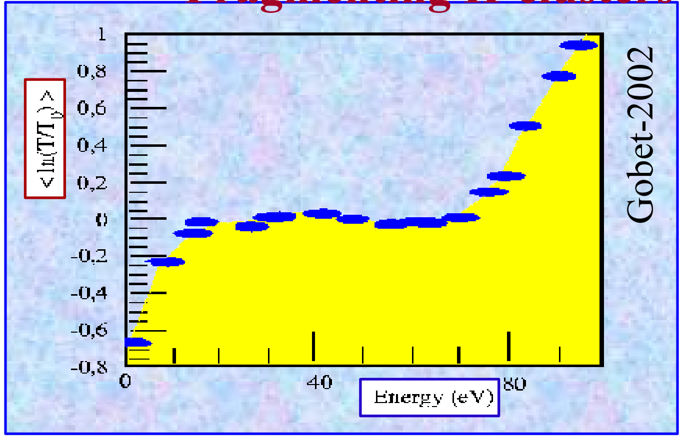

T decreasing with E

- IV-

Negative heat capacities

Abnormal energy fluctuations

Multifragmentation

Heat capacity from energy fluctuation

- Canonical: fluct. E_{tot}

  $Z_{tot} = Z_k Z_p$

  $\sigma_E^2 = \sigma_k^2 + \sigma_p^2$

  $\partial^2_\beta \log Z_i = C_i/\beta^2 = \sigma_i^2$

- Microcanonical: fluct. partial energy

  $W_{tot} = W_k \otimes W_p$

  $\sigma_E^2 = 0, \quad \sigma_k^2 = \sigma_p^2$

  $\partial^2_\beta \log W_{tot} = -1/CT^2 = f(\sigma_k^2)$

-

PC, Gulminelli NPA(1999)

Heat capacity from energy fluctuation

- Canonical: fluct. E_{tot}

  $Z_{tot} = Z_k Z_p$

  $\sigma_E^{\,2} = \sigma_k^{\,2} + \sigma_p^{\,2}$

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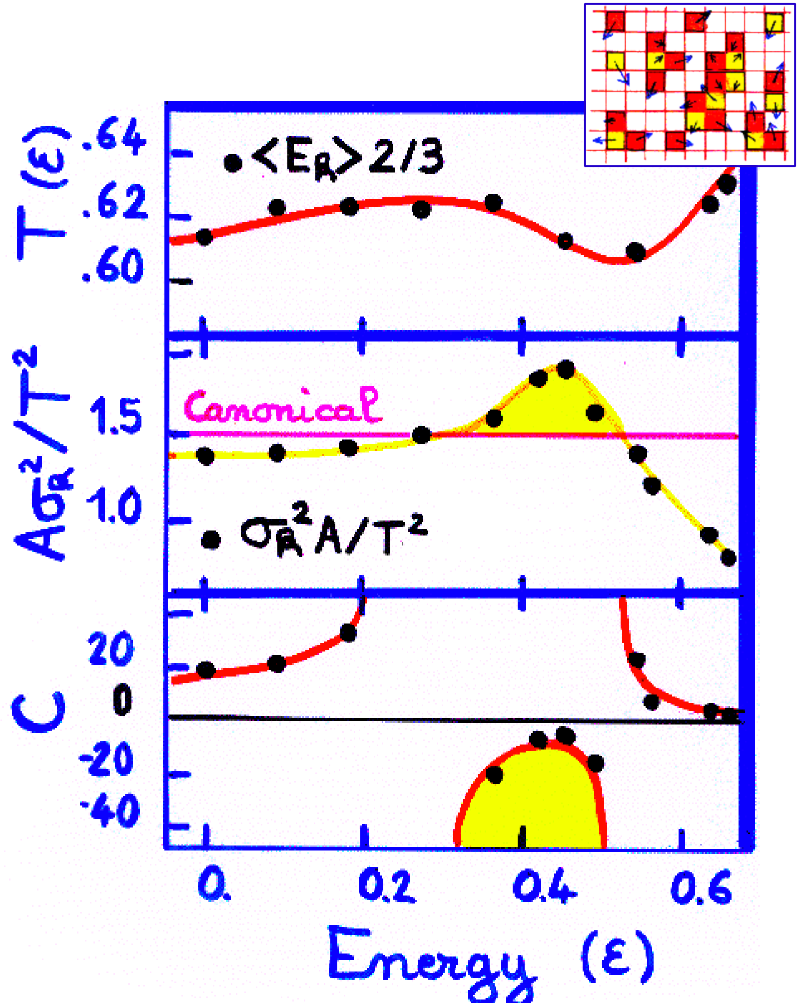

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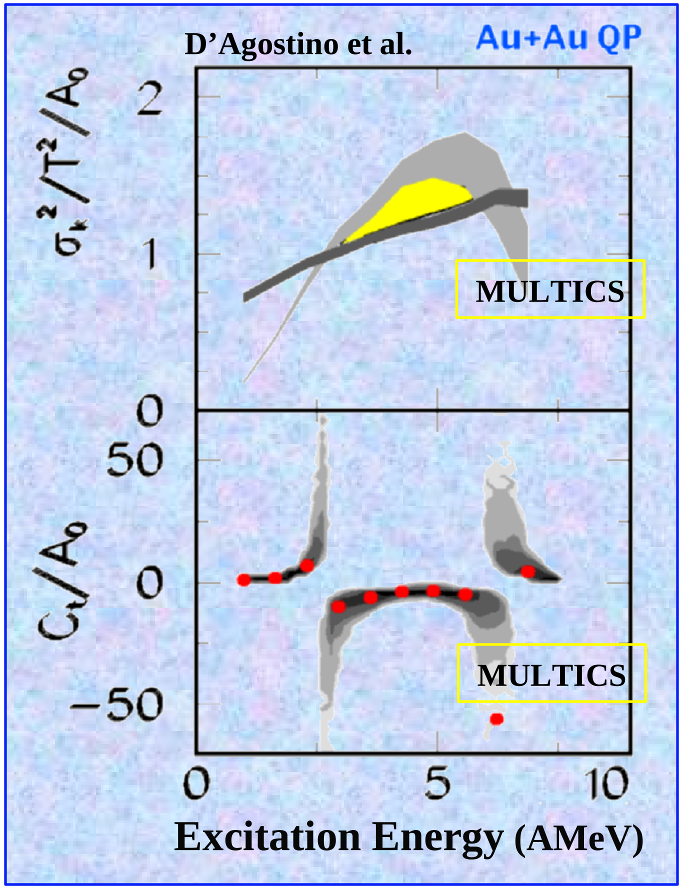
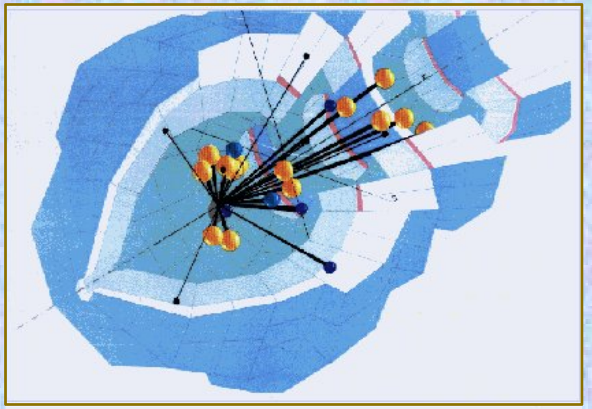
PC, Gulminelli NPA(1999)

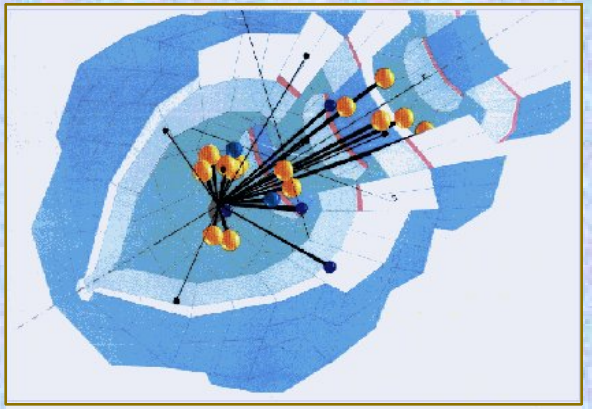


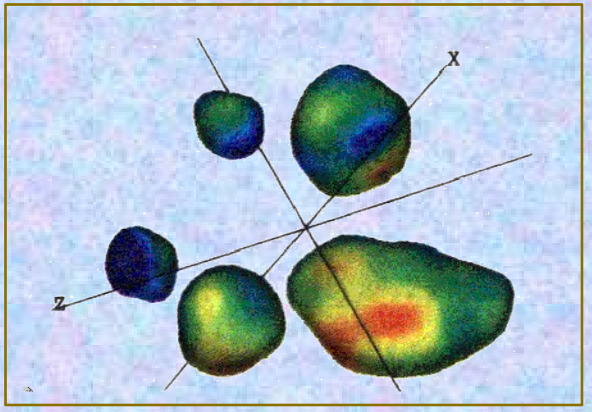
Multifragmentation experiment



- Sort events in E* (Calorimetry)

Multifragmentation experiment





- **Sort events in E* (Calorimetry)**
- **Reconstruct a freeze-out partition**

1- Primary fragments: m_i

2- Freeze-out volume: E_{coul}

- $E_2 = \Sigma_i B_i + E_{coul}$
- $E_1 = E^* - E_2$ => $\langle E_1 \rangle, \sigma_1$

3- Kinetic EOS: $T, C_1, \sigma^2_{can} = C_1 T^2$

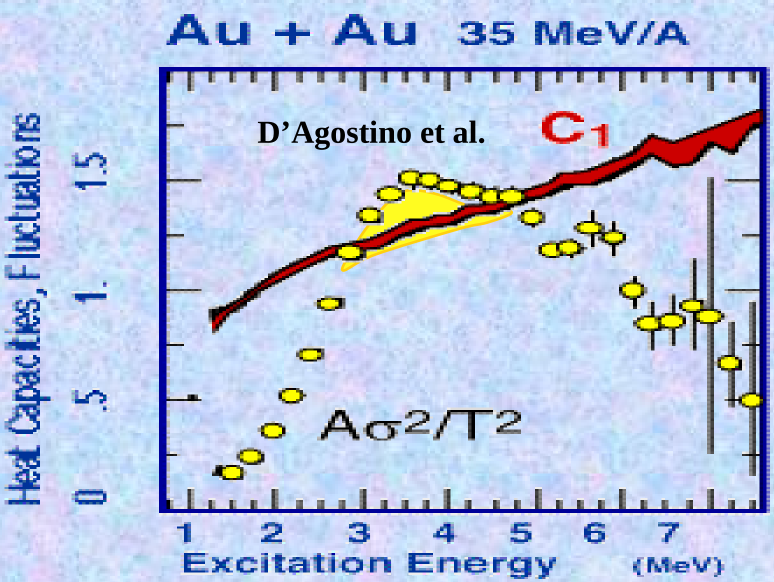
- $\langle E_1 \rangle = \langle \Sigma_i a_i \rangle T^2 + 3/2 \langle M-1 \rangle T$

- **Deduce C:**

Heat capacity from energy fluctuation

- Large fluctuations

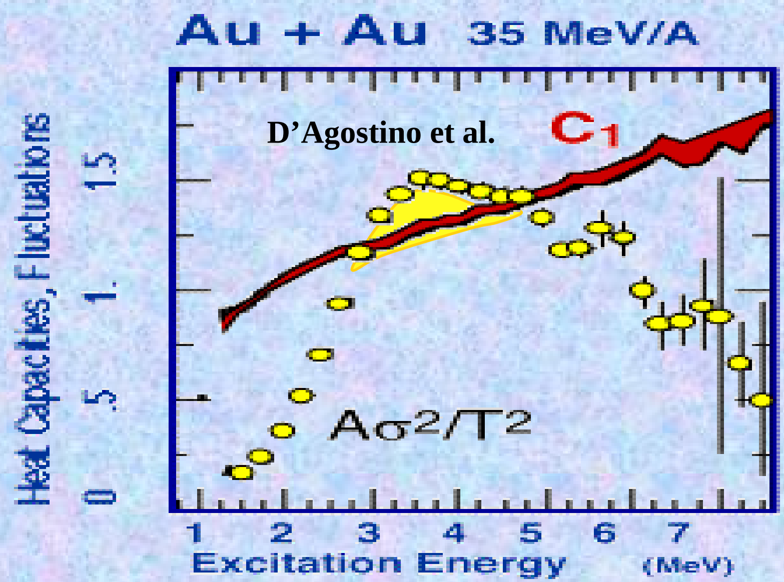
$$C = \frac{C_1^{\,2}}{C_1 - \sigma^2 / T^2}$$



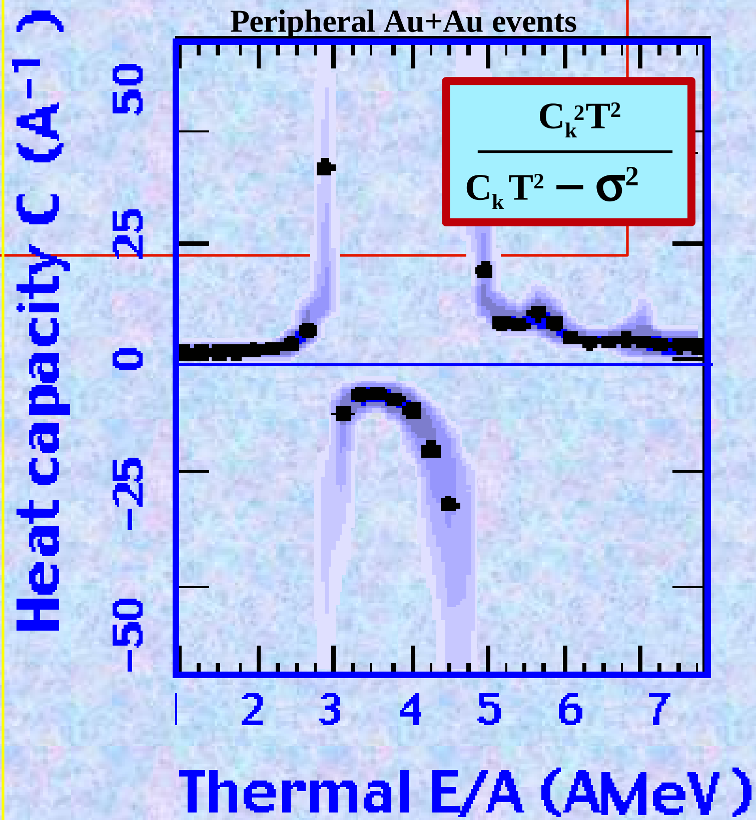
Heat capacity from energy fluctuation

- Large fluctuations

$$C = \frac{C_1^2}{C_1 - \sigma^2/T^2}$$



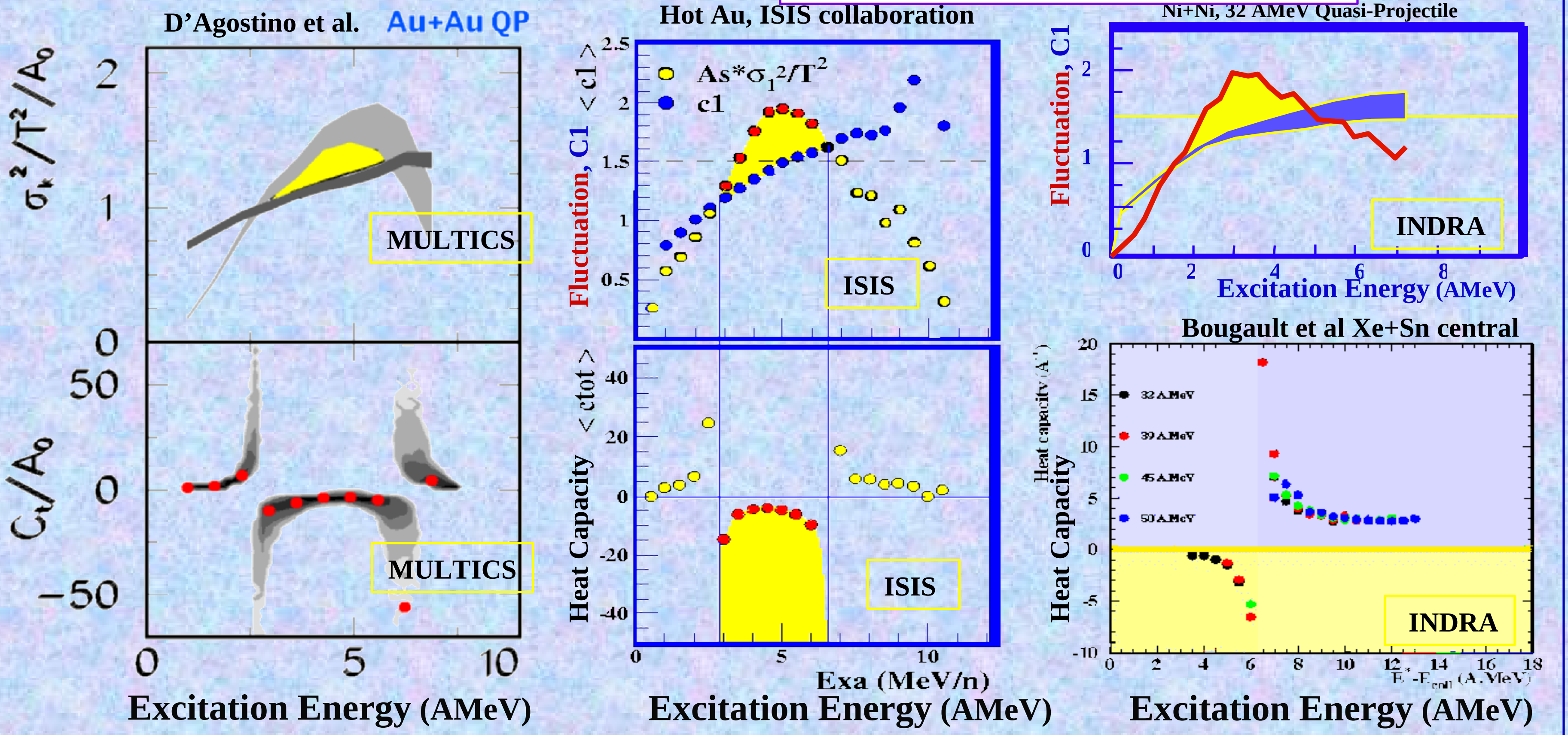
- Negative C



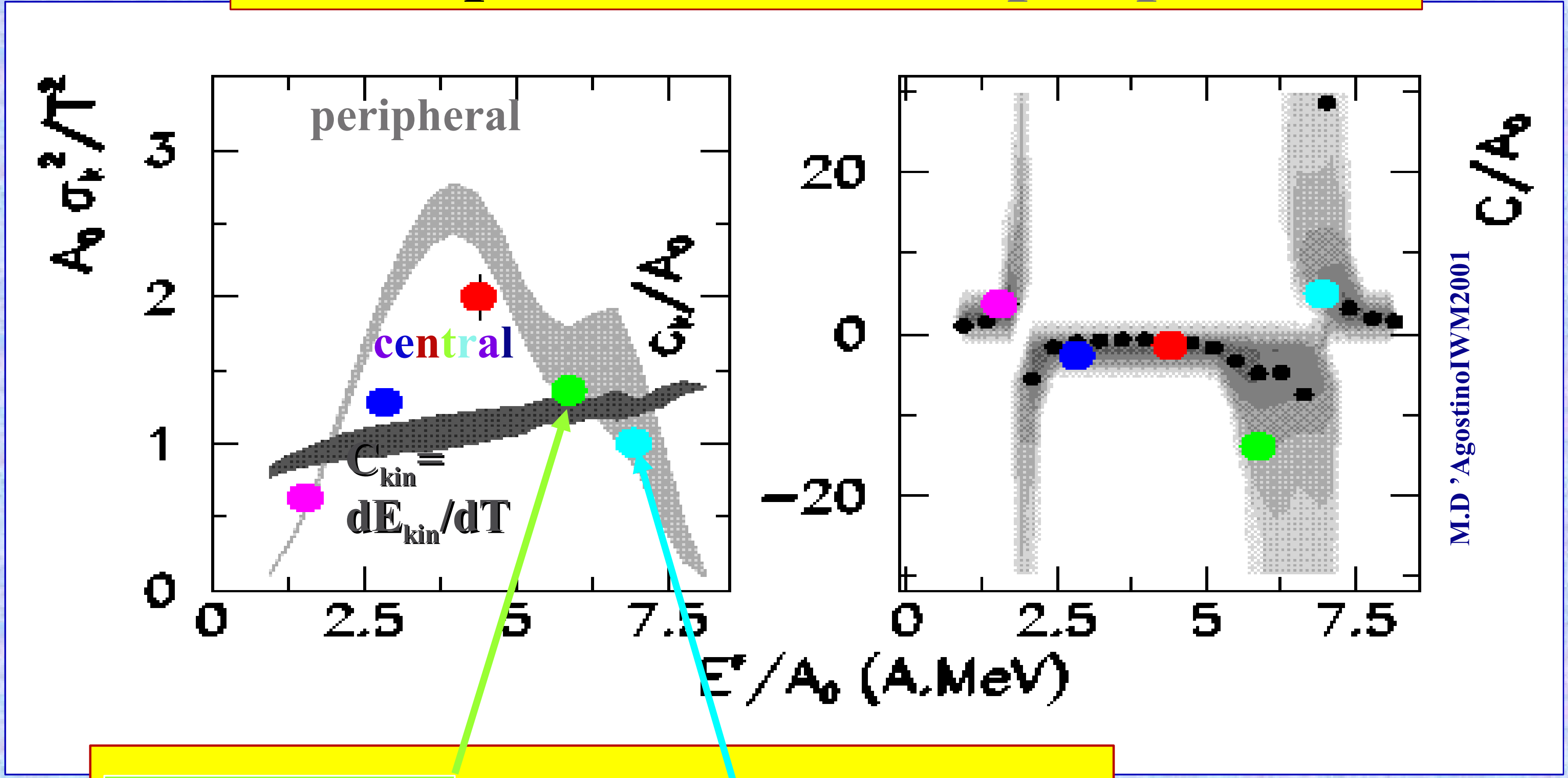
$$\frac{C_k^2 T^2}{C_k T^2 - \sigma^2}$$

Heat capacity from energy fluctuation

Multics E_1=2±0.3 E_2=6.5±0.7
Isis E_1=2.5 E_2=7.
Indra E_2=6.±0.5



Comparison central / peripheral



Au+Au 35 E_F=1

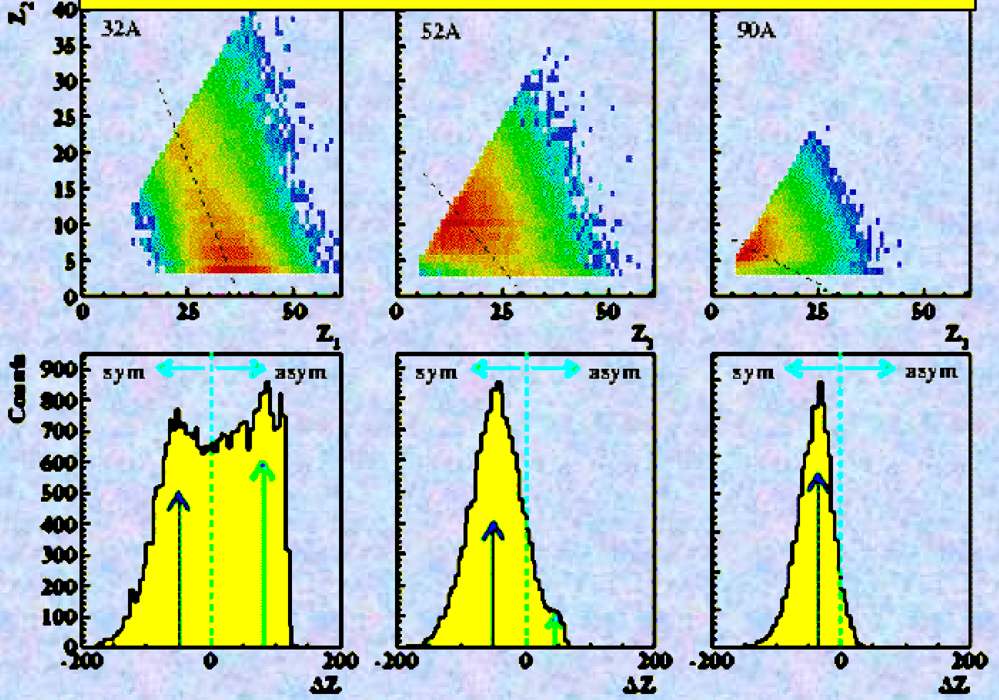
Au+Au 35 E_F=0

Up to ≈ 35 A.MeV the flow ambiguity is a small effect

Multifragmentation of nuclei

Bimodality in event distribution

Largest fragment/second largest

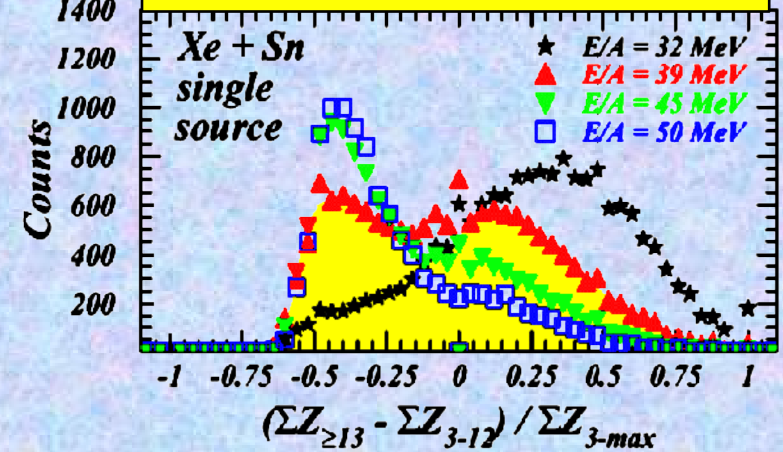


Ni+Au INDRA data

O.Lopez 2001

Distribution of events.

Charge big - small



Xe+Sn INDRA data

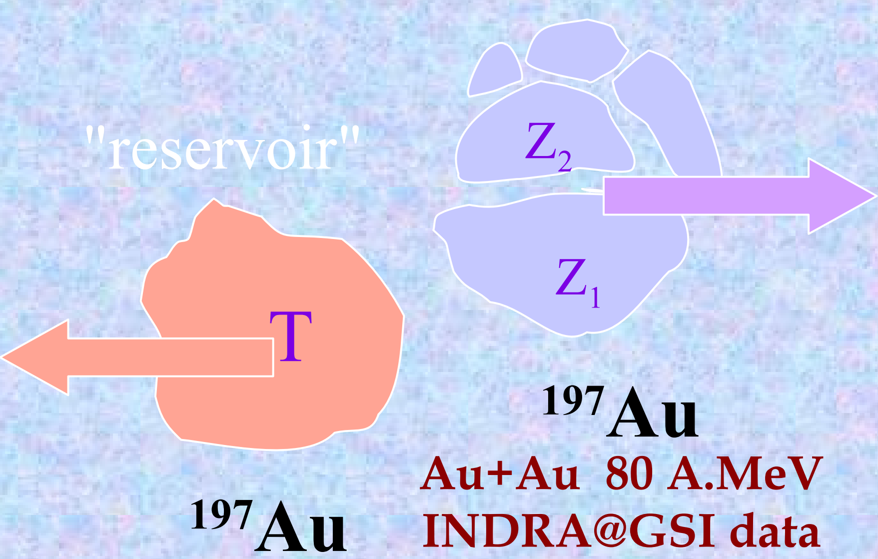
B. Borderie, 2001

Multifragmentation of nuclei

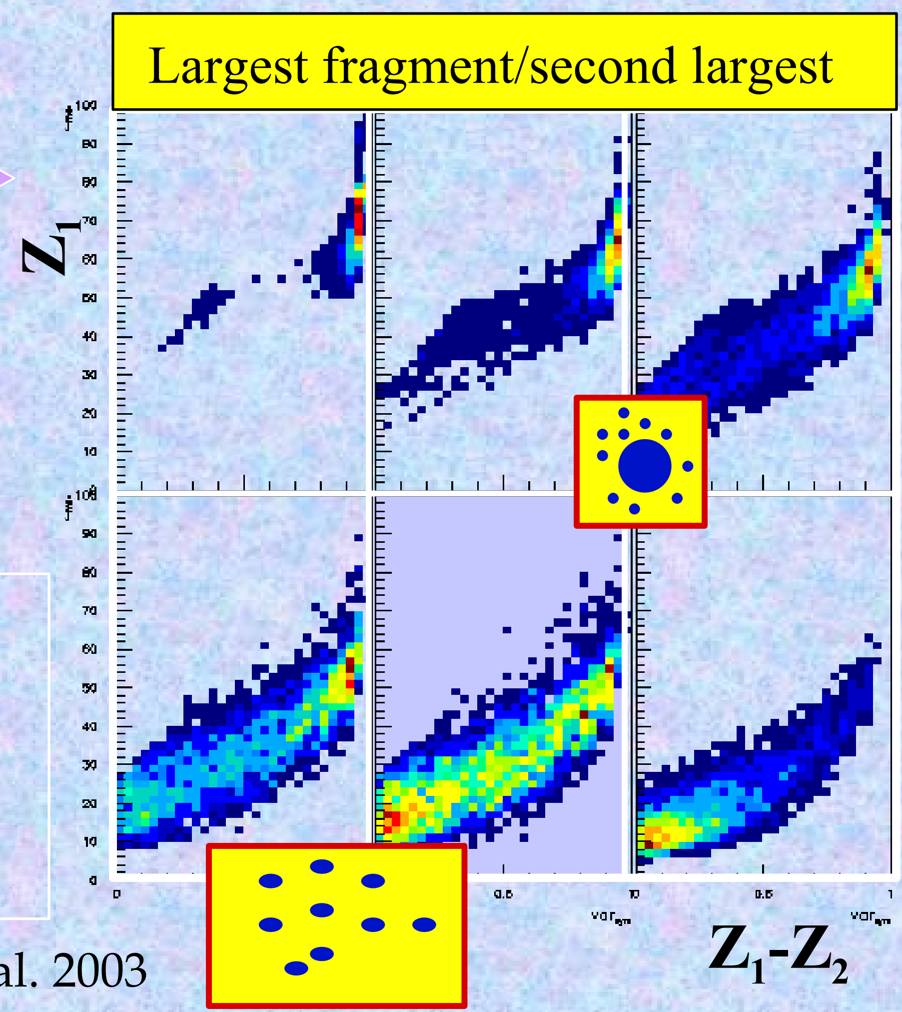
Bimodality in event distribution

Multifragmentation of nuclei

Bimodality in event distribution



Au+Au 80 A.MeV
INDRA@GSI data



- Order parameter
  - Z_1-Z_2 (big-small)
    - cf

O.Lopez et al. 2001, B.Tamain et al. 2003

- V -

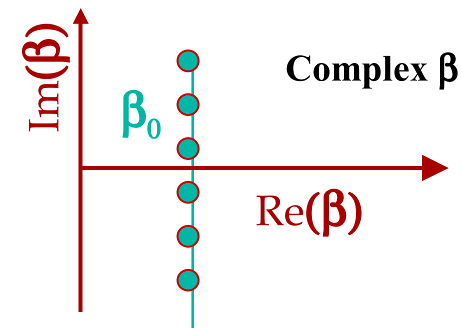
Conclusion

Negative heat capacities

Bimodality, fluctuations

1st order in finite systems

PC & Gulminelli Phys A (2003)

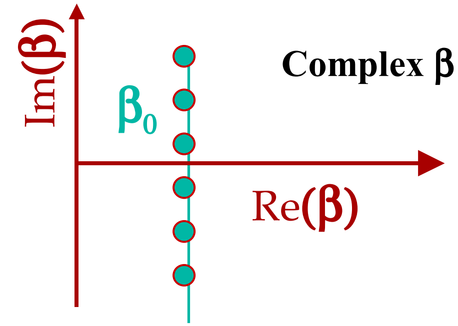


- **Order param. free** (canonical)
  - Zeroes of Z reach real β axis

Yang & Lee Phys Rev 87(1952)404

1st order in finite systems

PC & Gulminelli Phys A (2003)



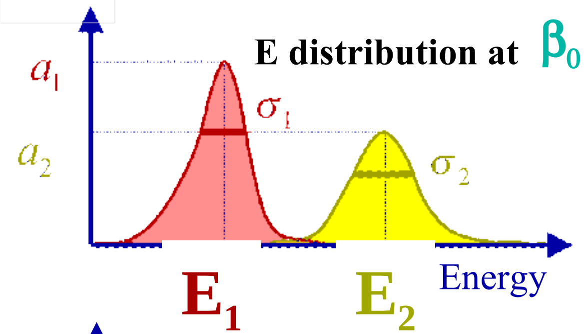
Order par. free (canonical)

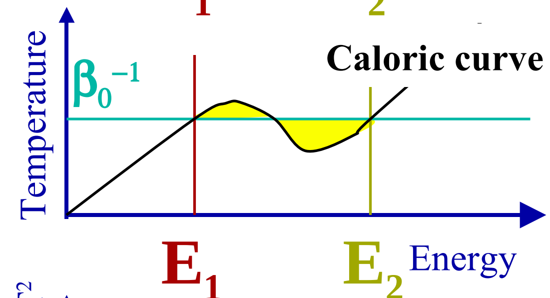
- Zeroes of Z reach real axis

  Yang & Lee Phys Rev 87(1952)404

- Bimodal E distribution ($P_\beta(E)$)

  K.C. Lee Phys Rev E 53 (1996) 6558





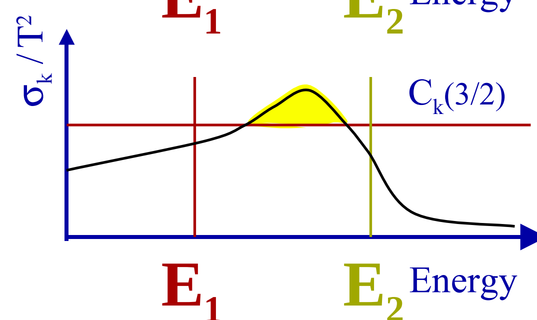
Order par. fixed (microcanonical)

- Back Bending in EOS (T(E))

  K. Binder, D.P. Landau Phys Rev B30 (1984) 1477

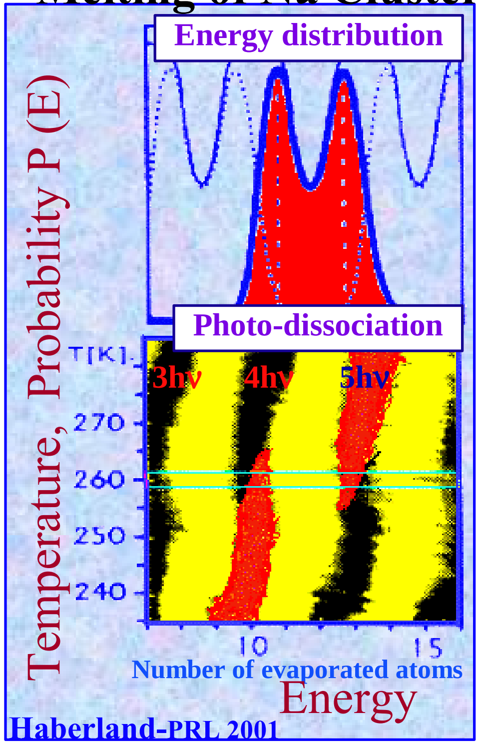
- Abnormal fluctuation ($\sigma_k(E)$)

  J.L. Lebowitz (1967), PC & Gulminelli, NPA 647(1999)153

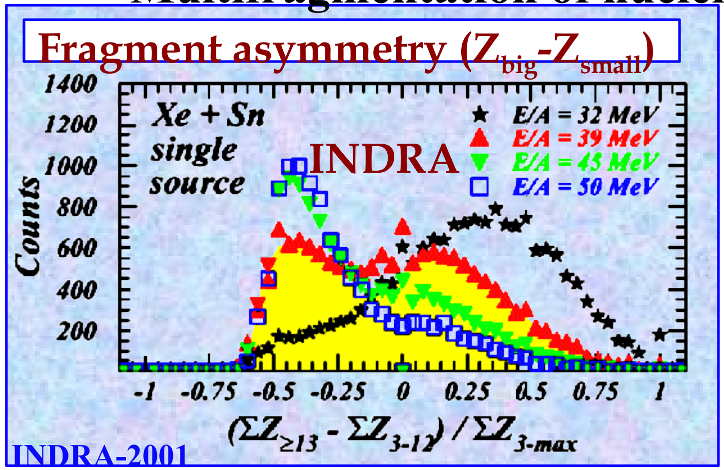


Seen in experiments

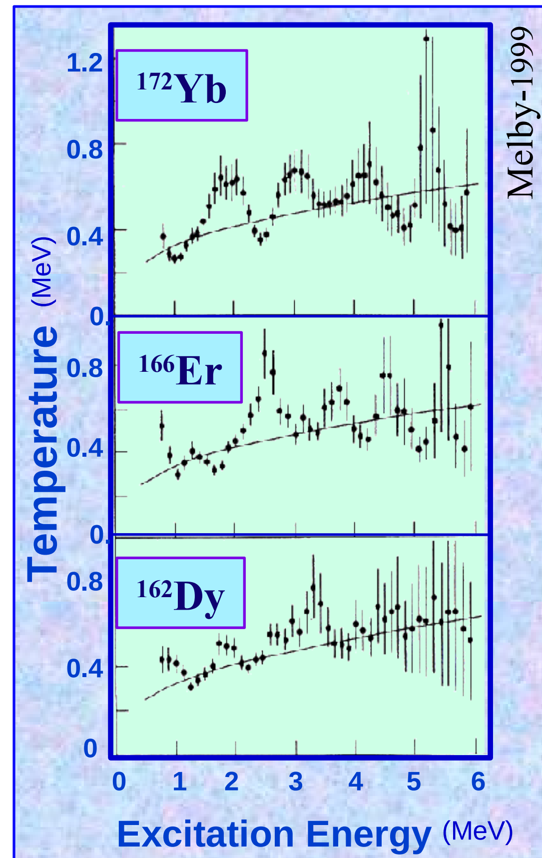
Melting of Na Cluster



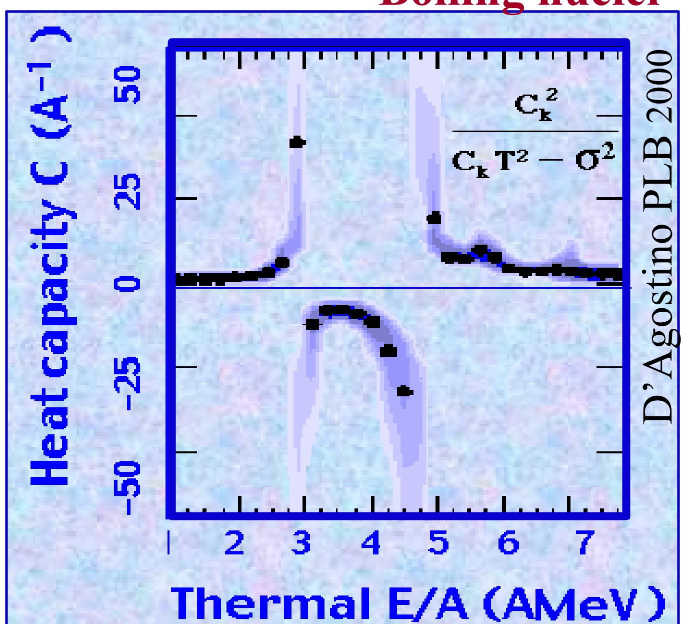
Multifragmentation of nuclei



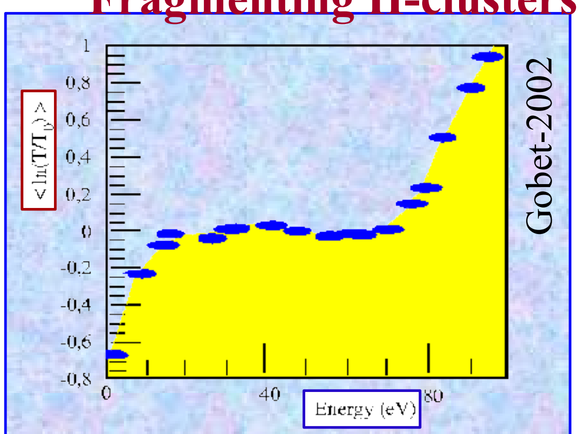
Nuclear superfluidity



Boiling nuclei



Fragmenting H-clusters

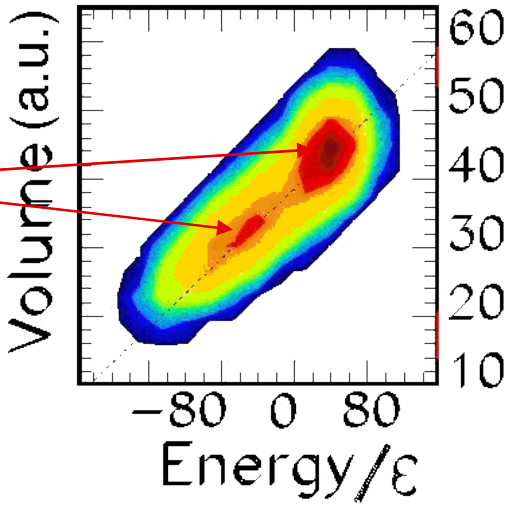


Liquid-gas

Isobare ensemble

$P^{(i)} \propto \exp\text{-}\lambda V^{(i)}$

- **Phase transition and bimodality**

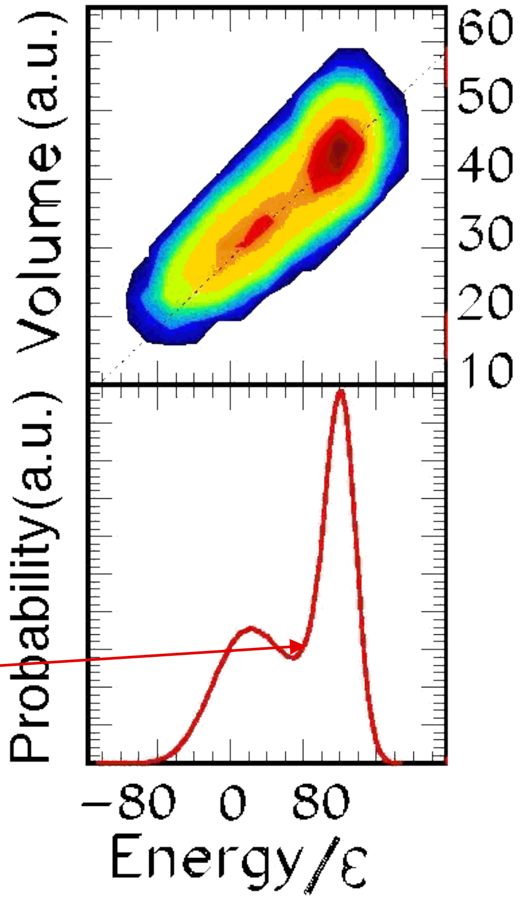


Liquid-gas

Isobare ensemble

$P^{(i)} \propto \exp -\lambda V^{(i)}$

- **Phase transition and bimodality**
- **Negative heat capacity**

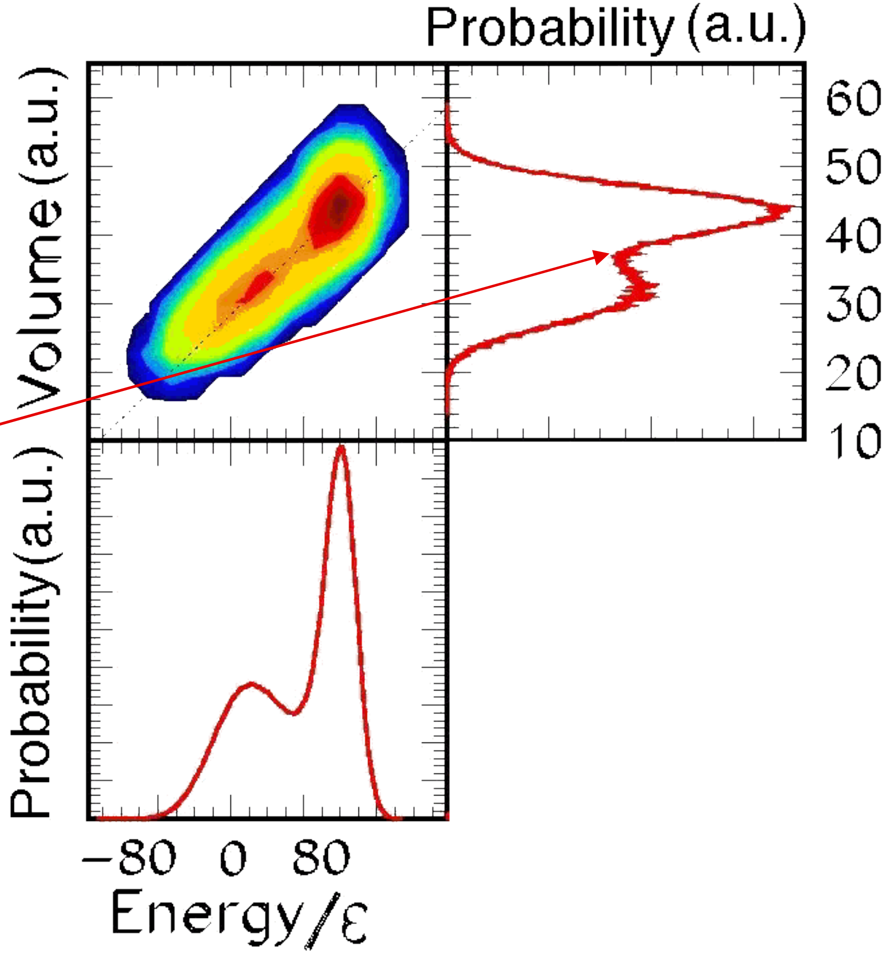


Liquid-gas

Isobare ensemble

$P^{(i)} \propto \exp{-\lambda V^{(i)}}$

- **Phase transition And bimodality**
- **Negative compressibility**
- **Negative heat capacity**

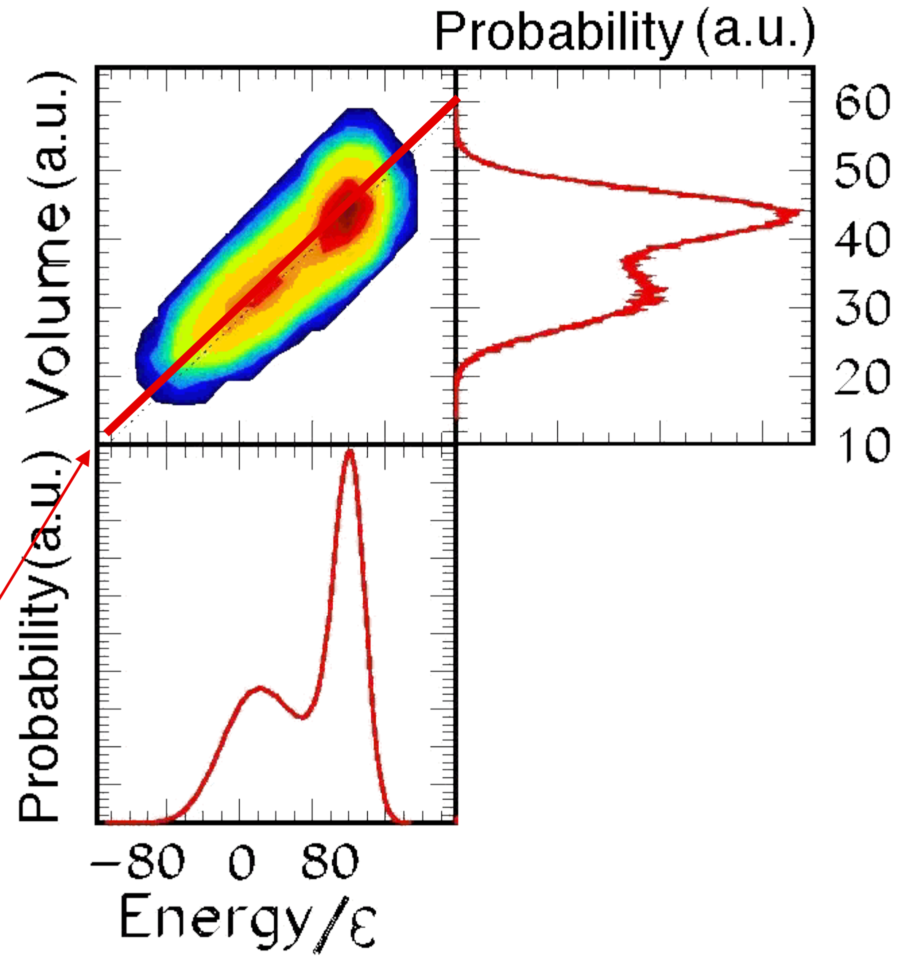


Liquid-gas

Isobare ensemble

$P^{(i)} \propto \exp -\lambda V^{(i)}$

- **Phase transition And bimodality**
- **Negative compressibility**
- **Negative heat capacity**
- **Order parameter**



Liquid-gas

Isobare ensemble

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- Phase transition And bimodality
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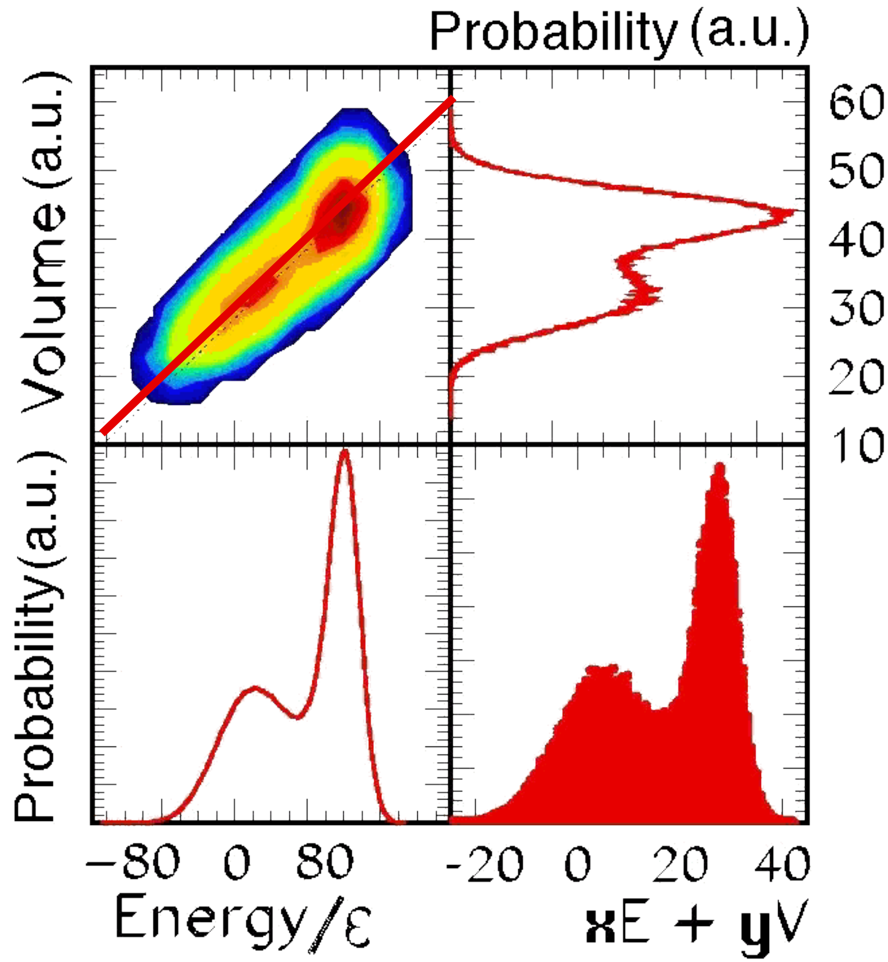
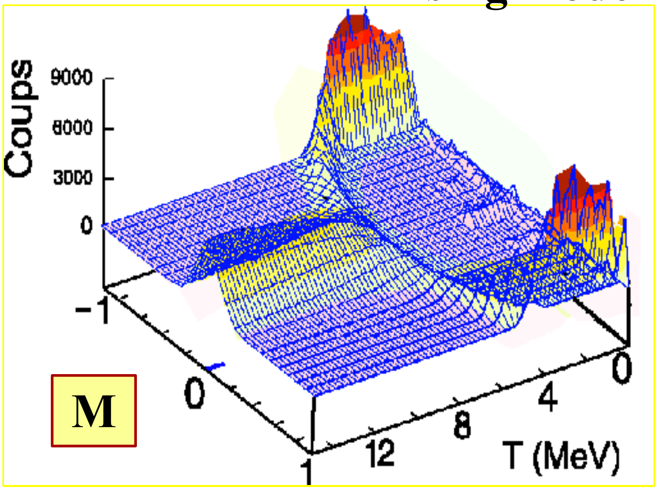


Bimodal distributions

Ising model



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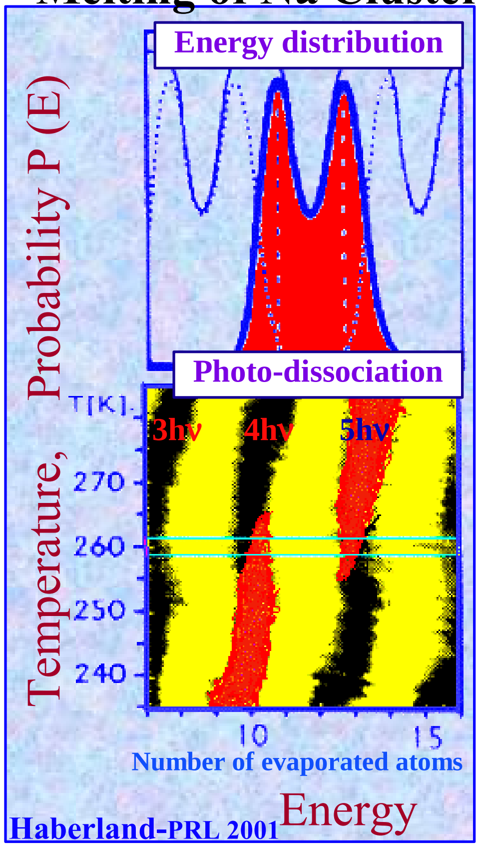

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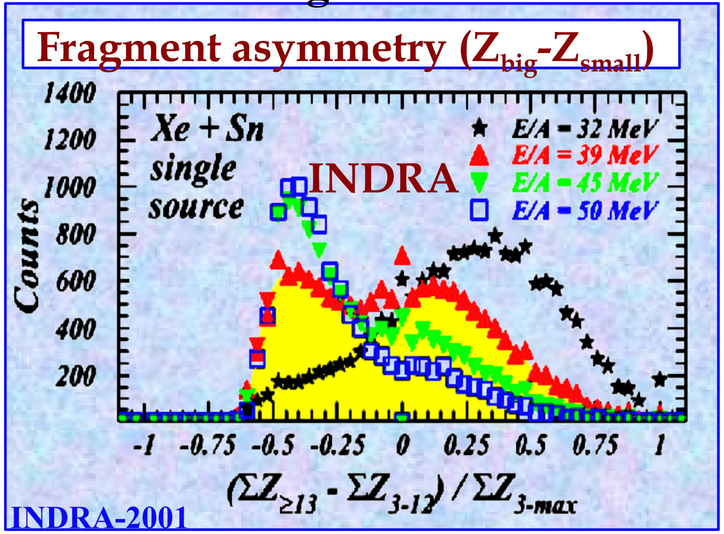
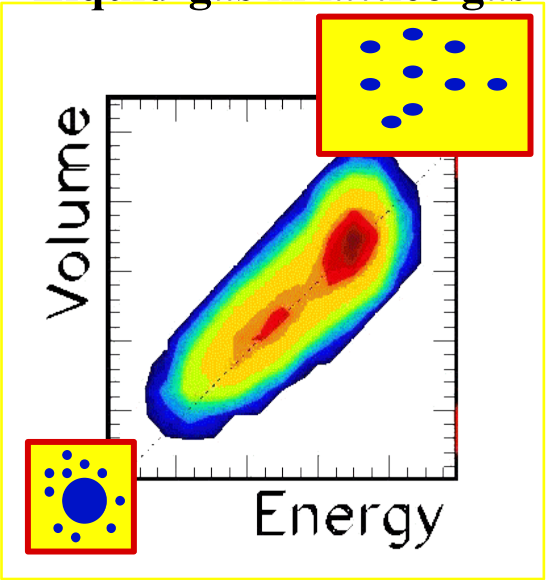
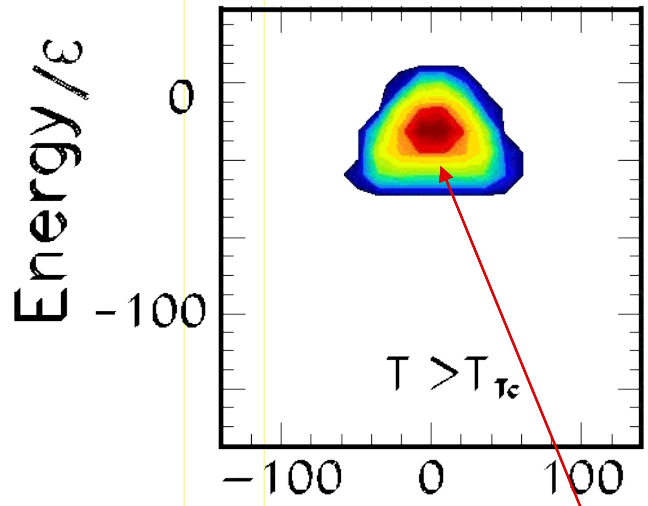


Liquid-gas in lattice-gas

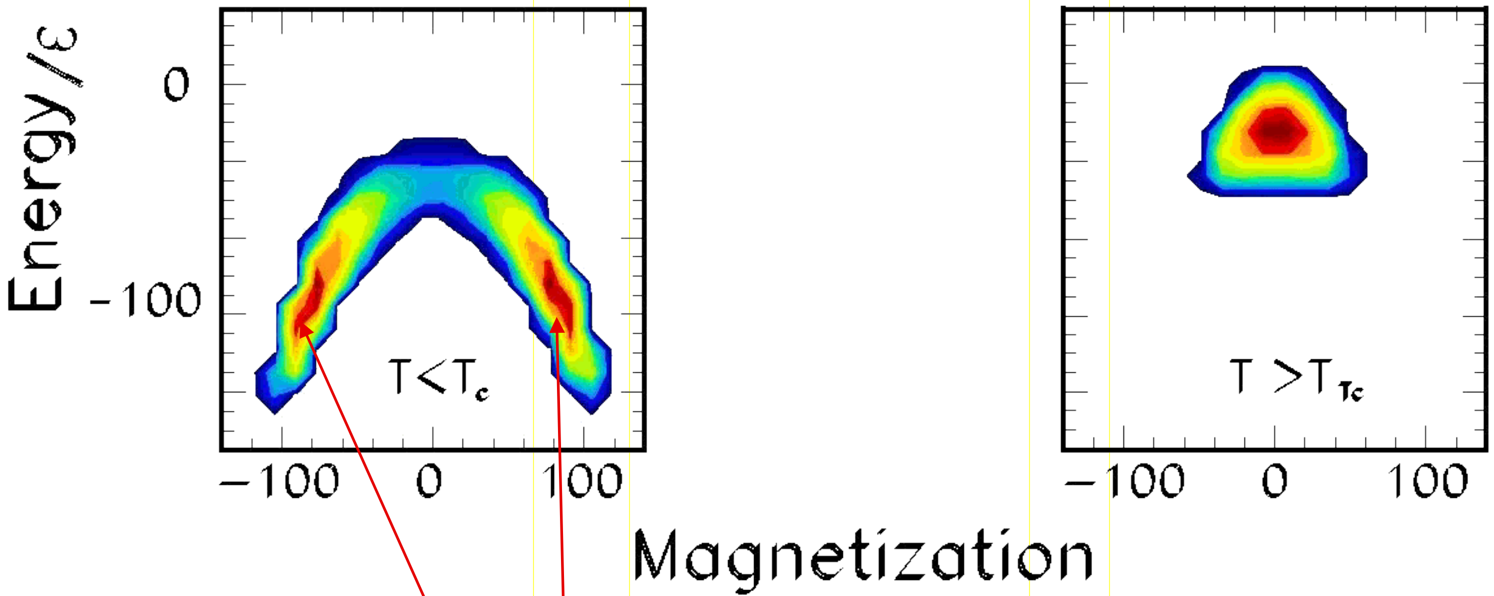


Magnetization in Ising Model



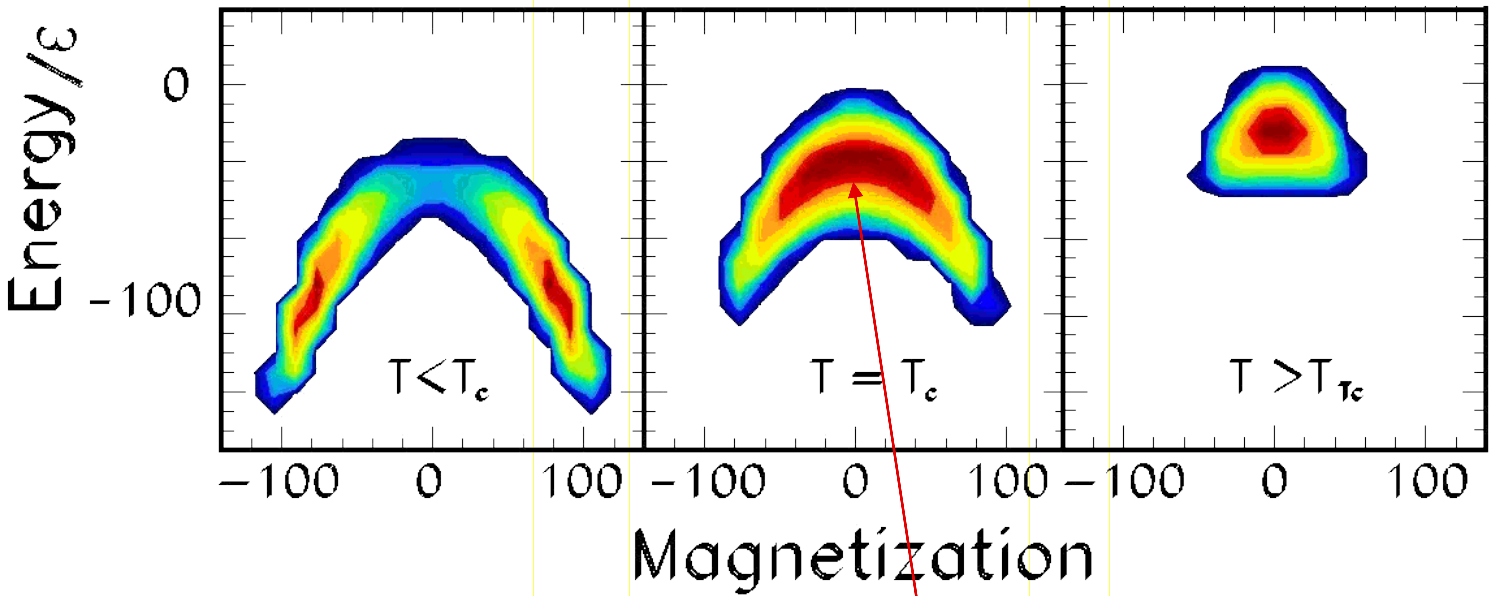
- One Phase

Magnetization in Ising Model



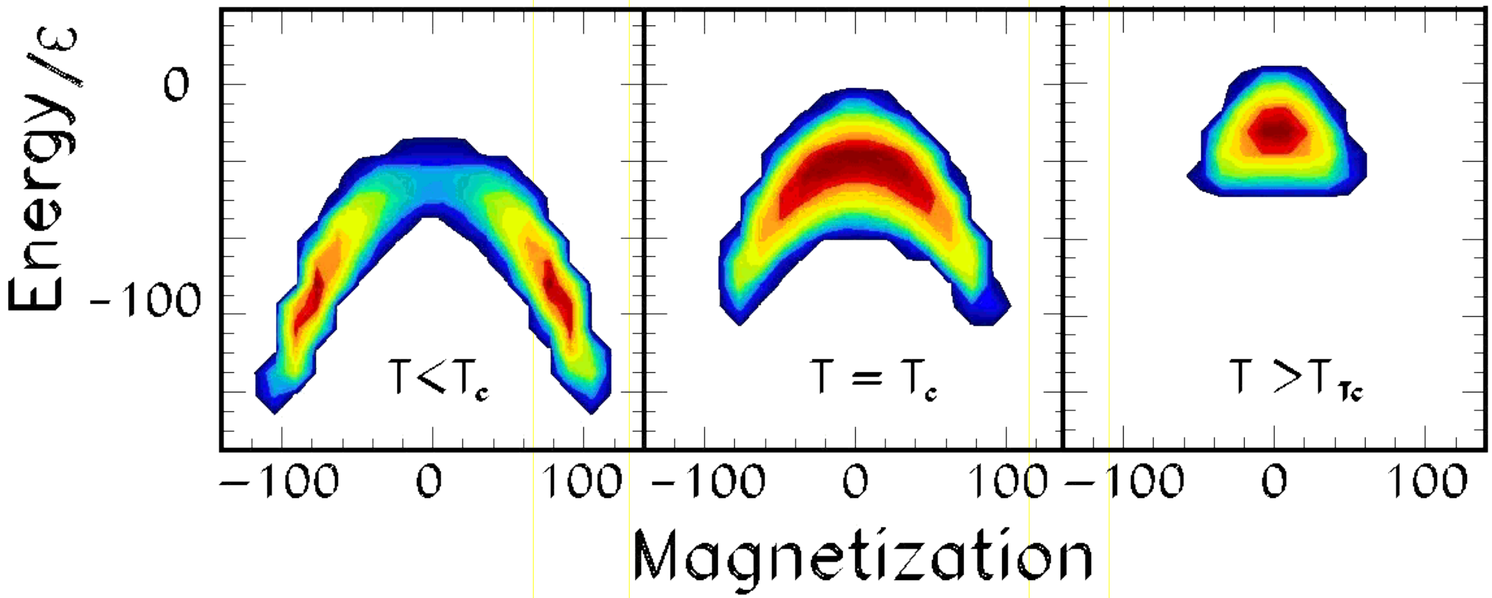
- Bimodal
  Two Phases
- One Phase

Magnetization in Ising Model



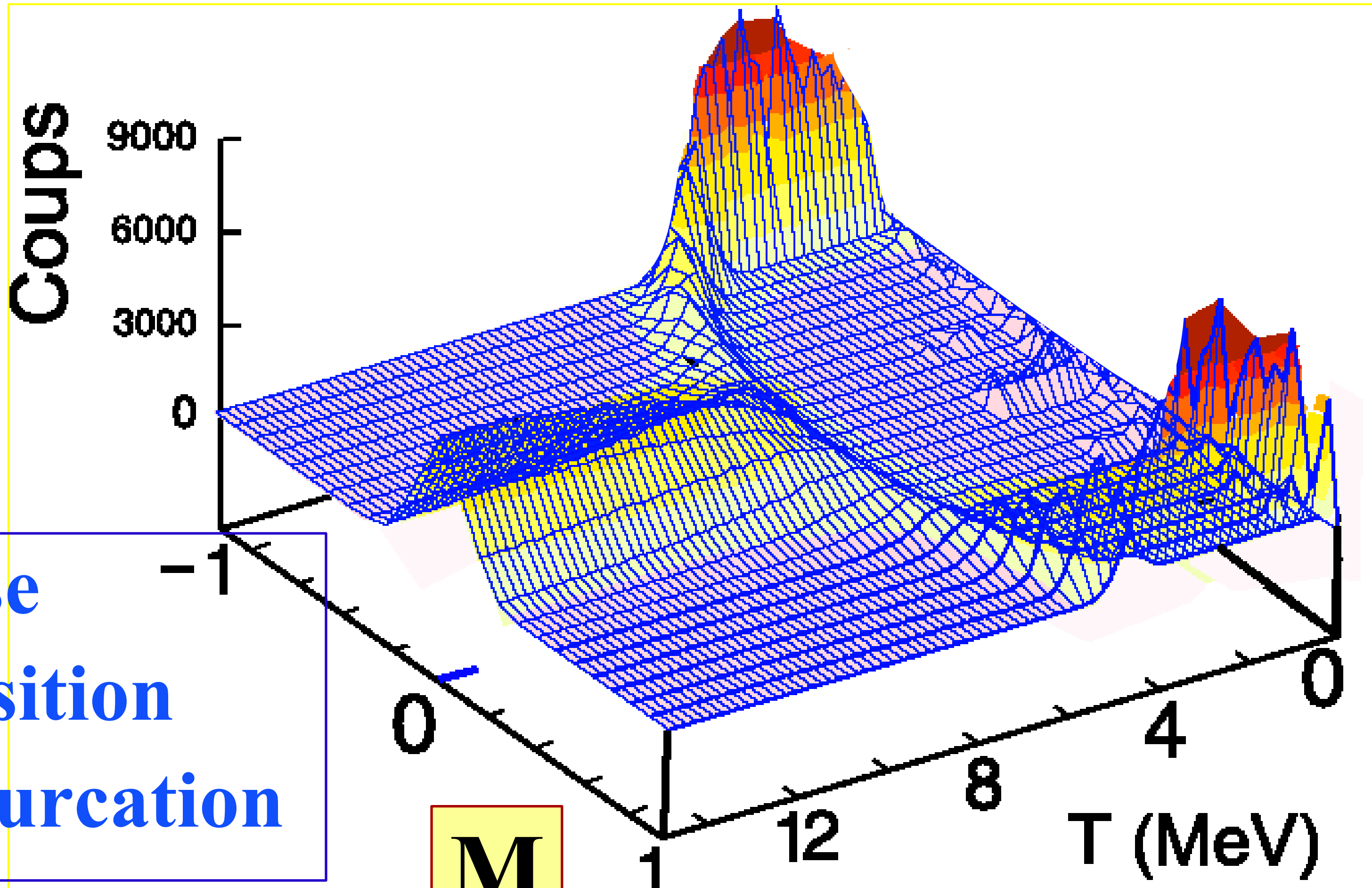
- Bimodal **Two Phases**
- Second Order
- One Phase

Magnetization in Ising Model



- Bimodal **Two Phases**
- Second Order
- One Phase

Magnetization in Ising Model

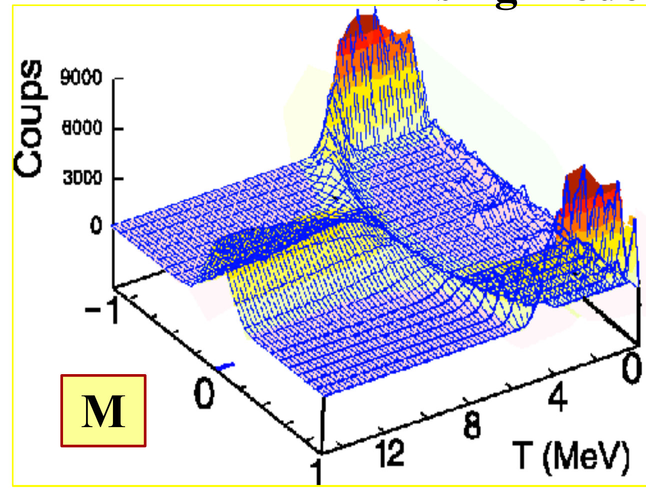


- **Phase transition a bifurcation**

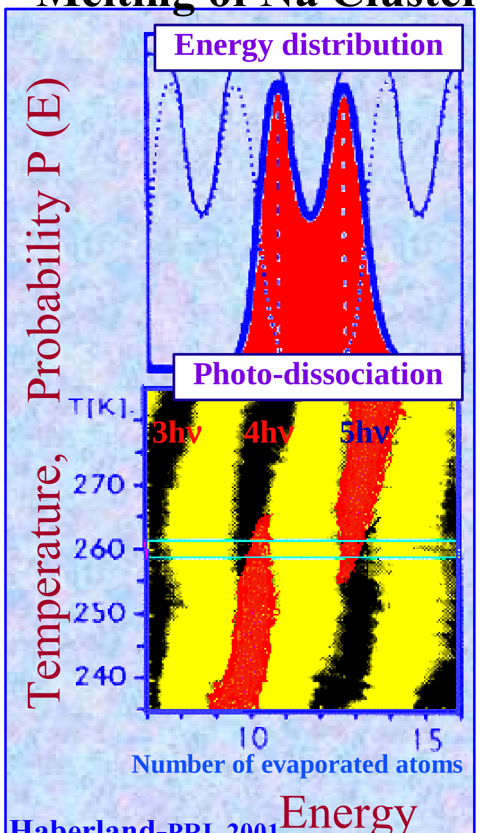
M

Bimodal distributions

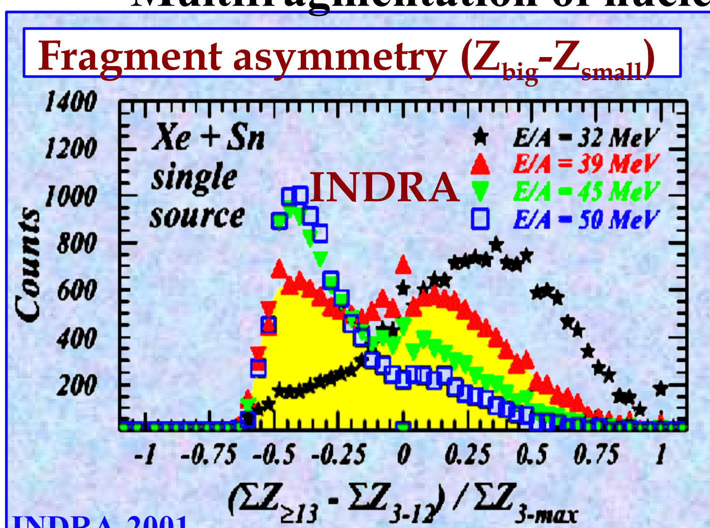
Ising model



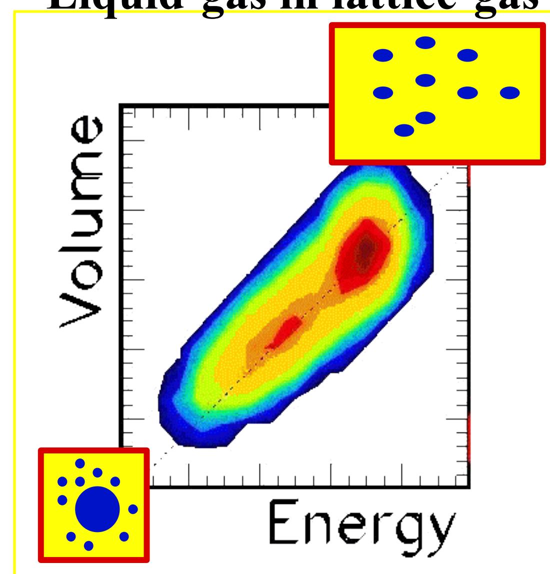
Melting of Na Cluster



Multifragmentation of nuclei

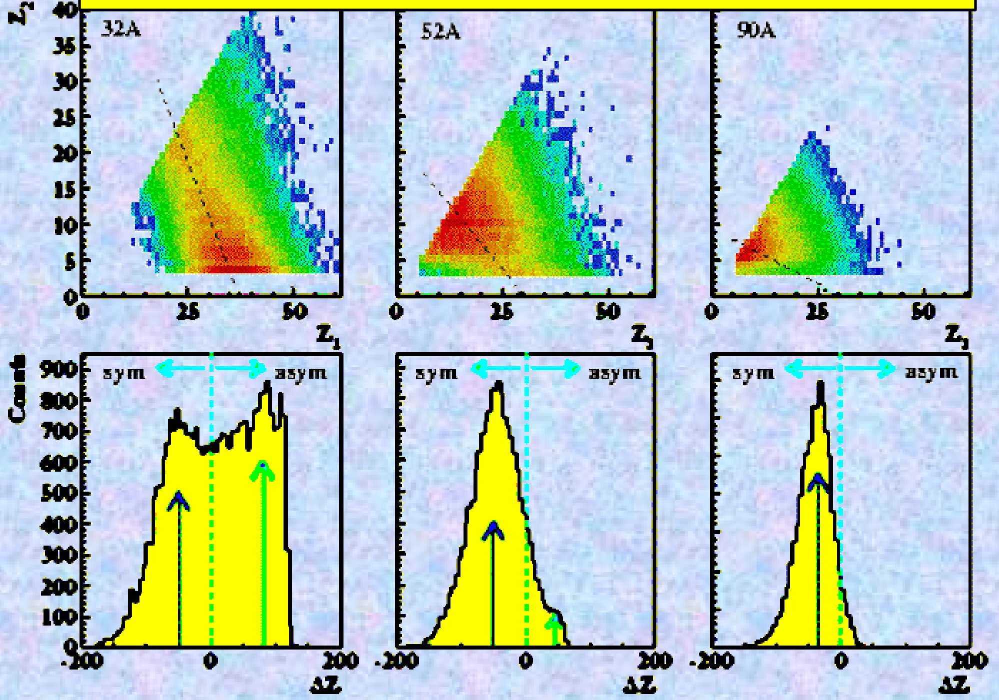


Liquid-gas in lattice-gas



Multifragmentation of nuclei
Bimodality in event distribution

Largest fragment/second largest

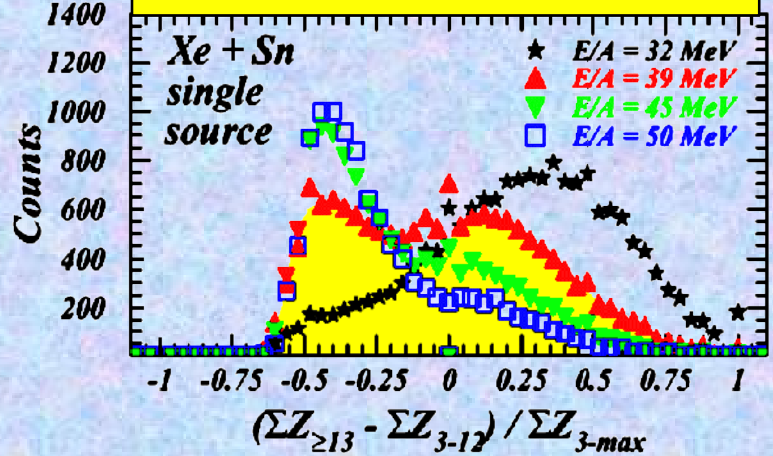


Ni+Au INDRA data

O.Lopez 2001

Distribution of events.

Charge big - small



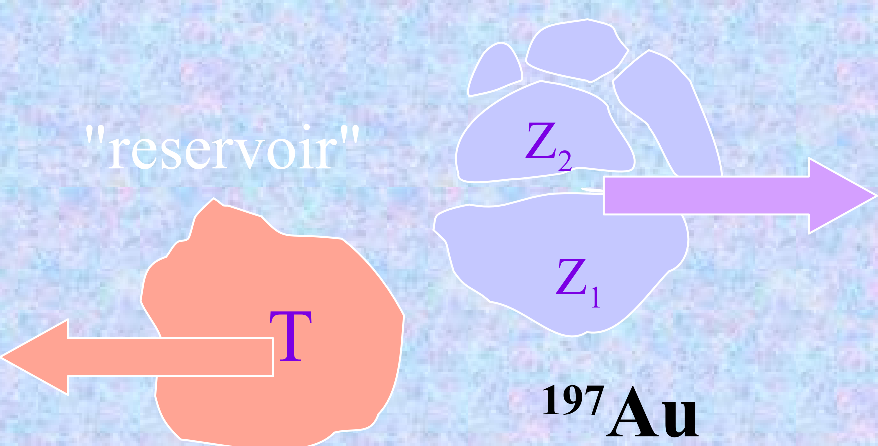
Xe+Sn INDRA data

B. Borderie, 2001

Multifragmentation of nuclei

Bimodality in event distribution

Multifragmentation of nuclei
Bimodality in event distribution

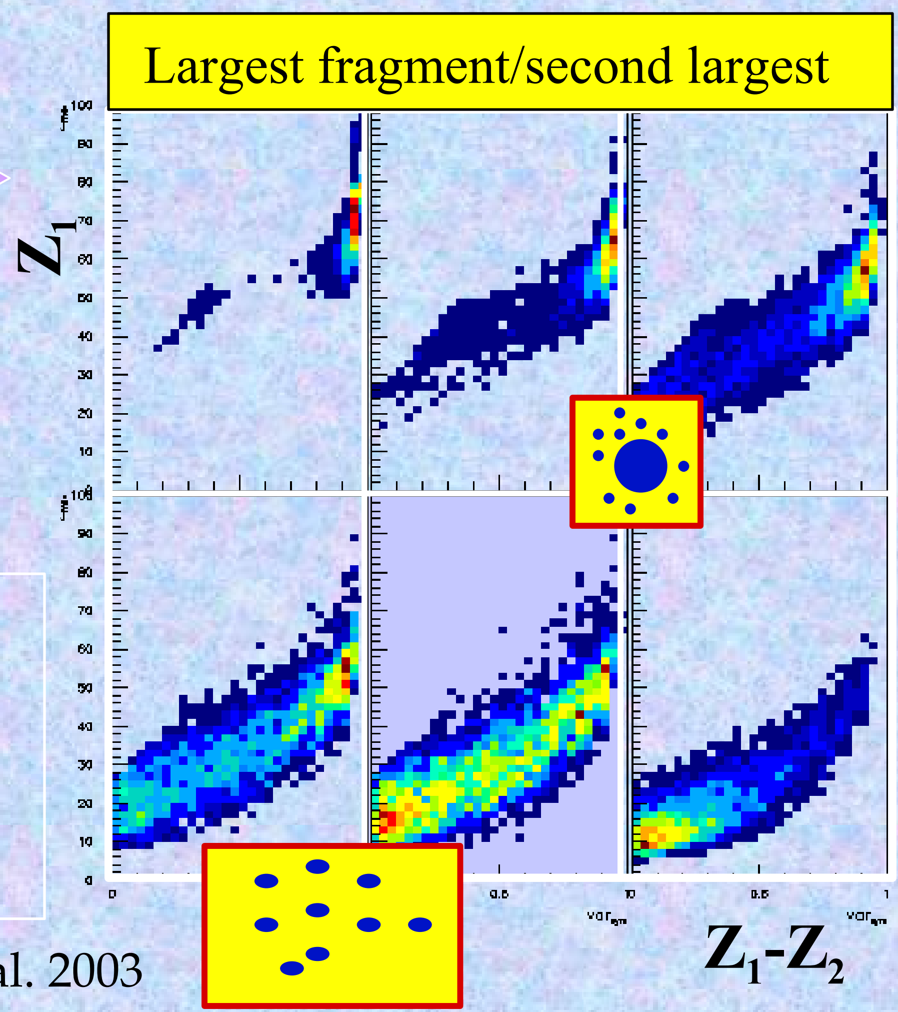


^{197}Au

^{197}Au

Au+Au 80 A.MeV
INDRA@GSI data

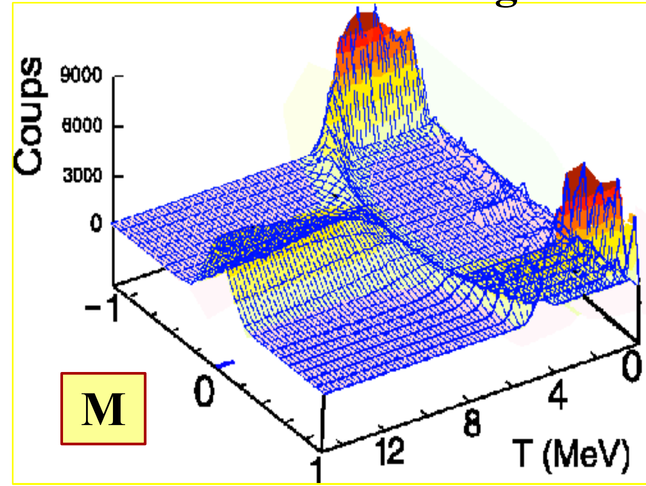
- Order parameter
  - Z_1-Z_2 (big-small)
    - cf



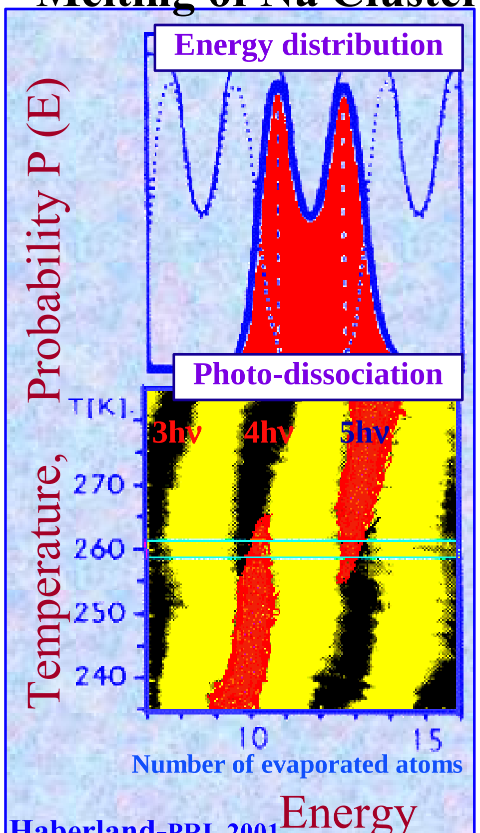
O.Lopez et al. 2001, B.Tamain et al. 2003

Bimodal distributions

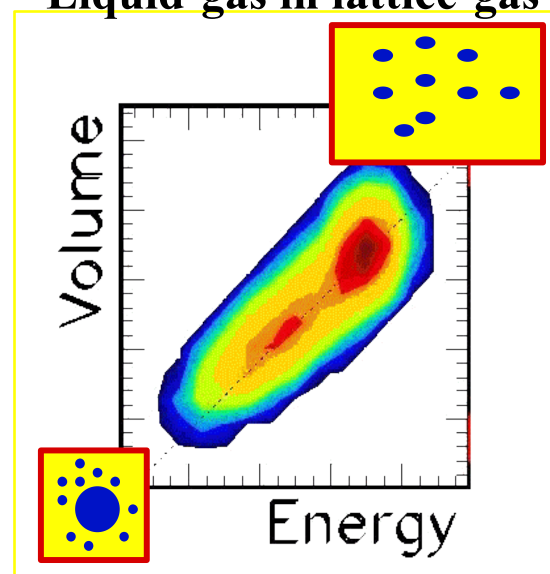
Ising model



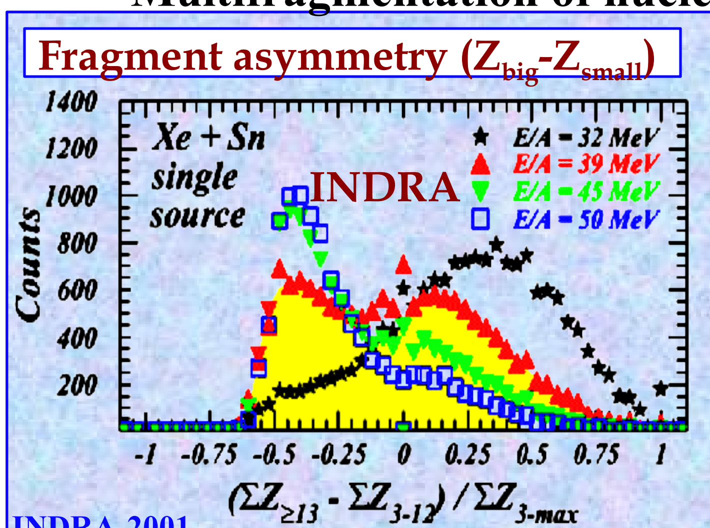
Melting of Na Cluster



Liquid-gas in lattice-gas

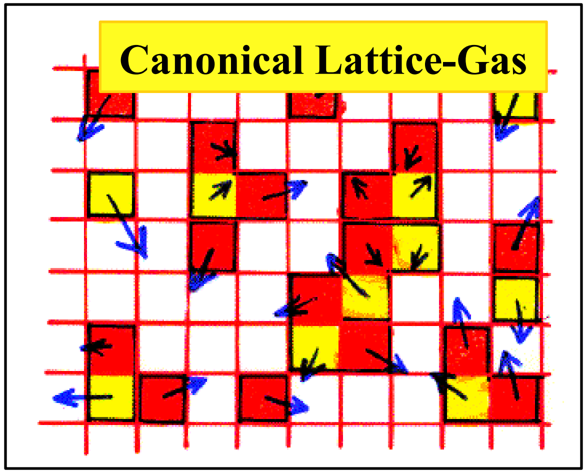


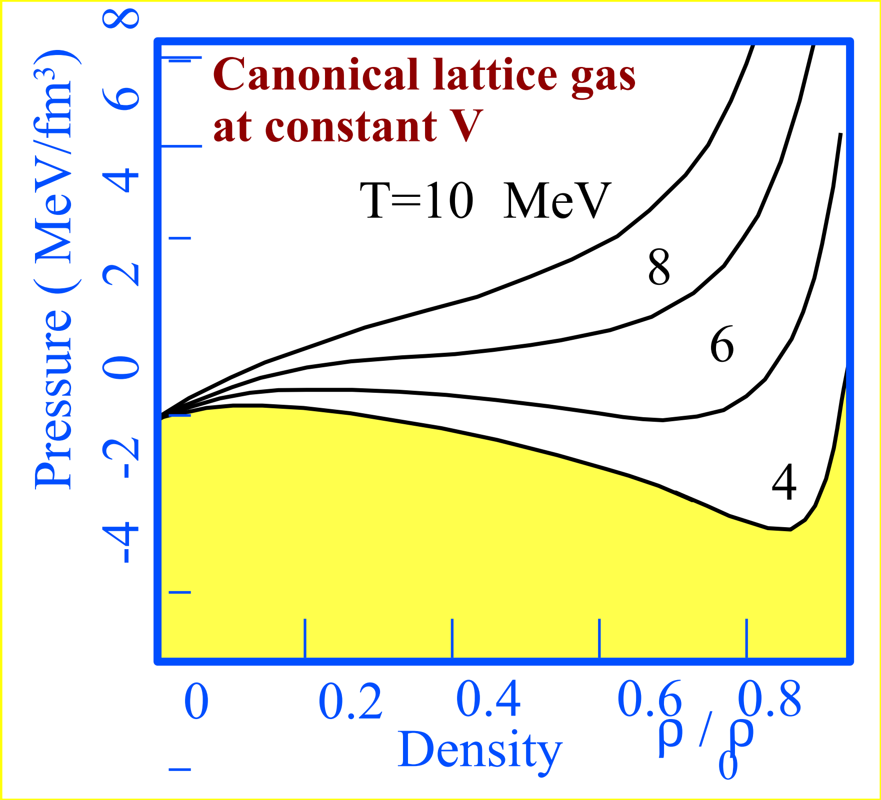
Multifragmentation of nuclei



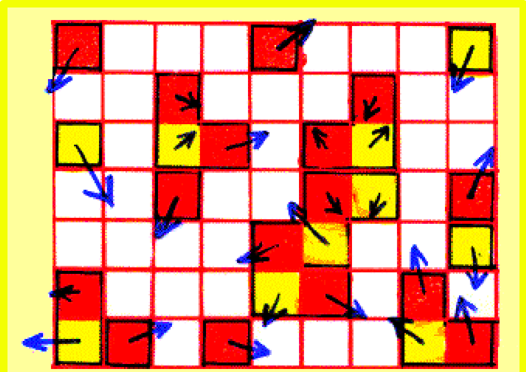
Density:
Order parameter:

- Usually V=cst
  - ("box")
  - Isochore ensemble
- **Negative compressibility**





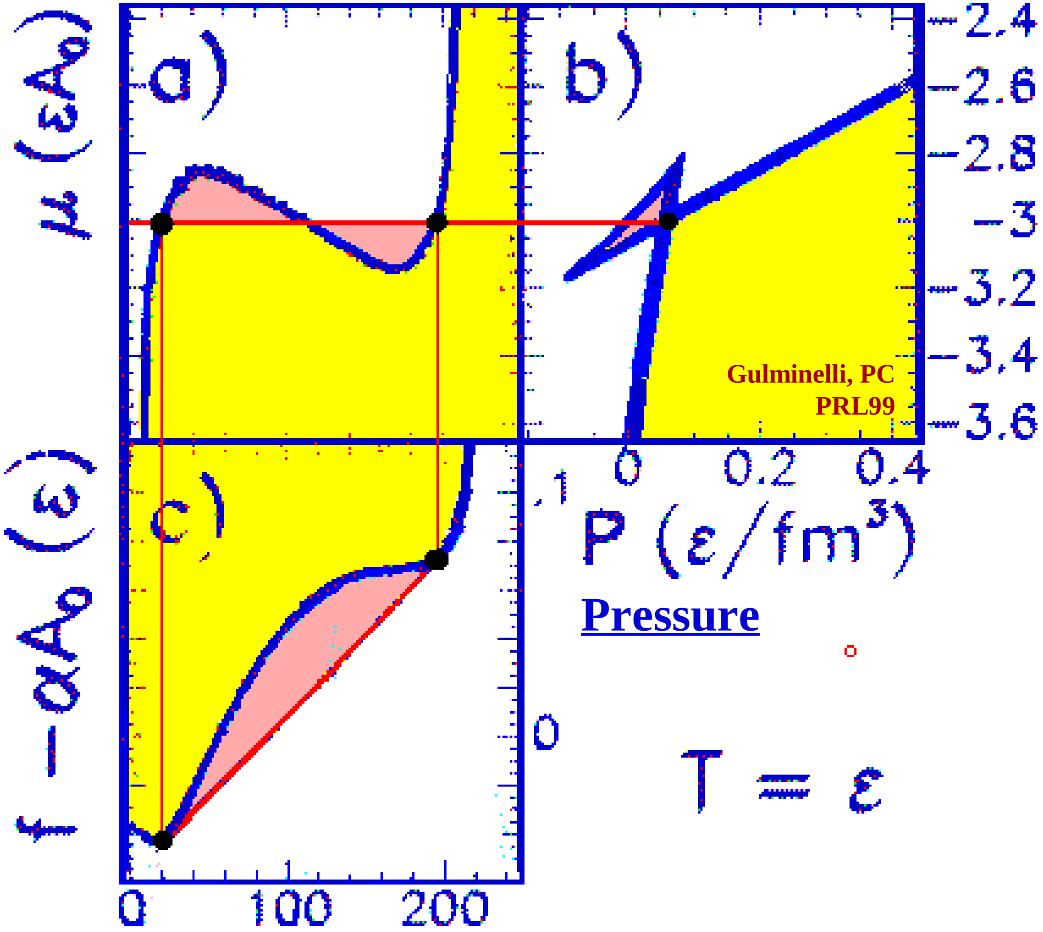
Generalization: inverted curvatures



Canonical Lattice-Gas

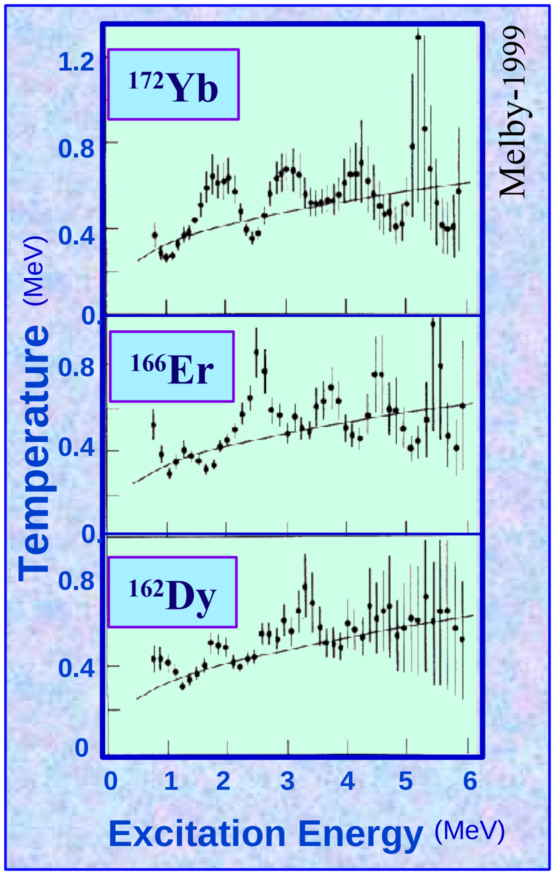
- **Negative Susceptibility and Compressibility**

Canonical Lattice-Gas



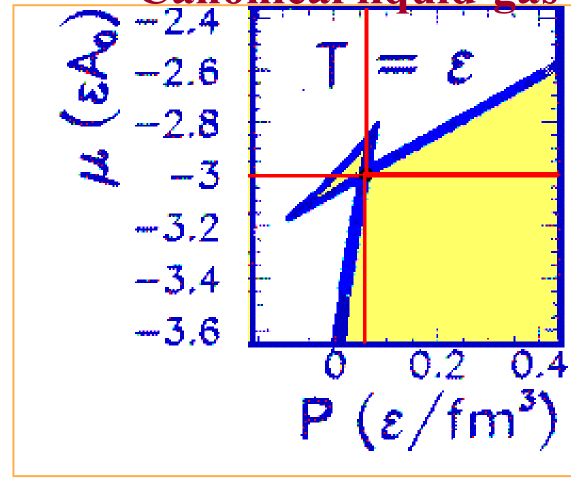
Abnormal curvatures

Nuclear superfluidity



Convex entropy

Canonical liquid-gas

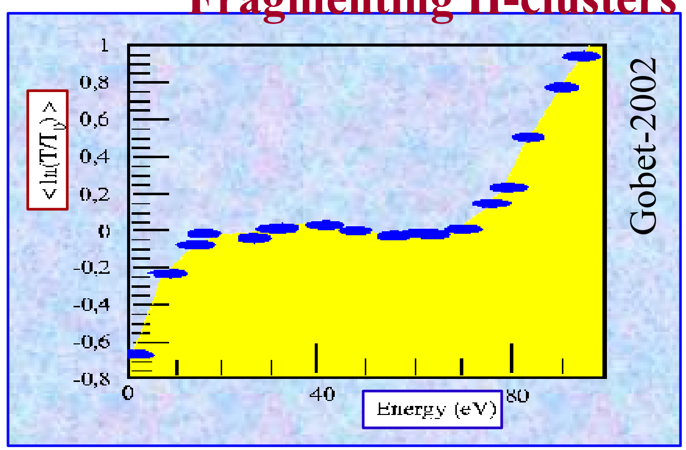


F. Gulminelli, Ph. Ch. PRL(1998)

Compressibility < 0

Susceptibility < 0

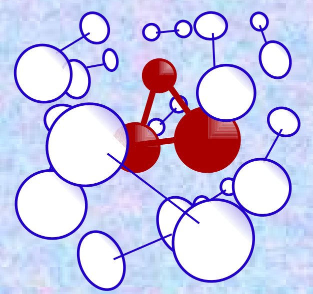
Fragmenting H-clusters

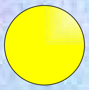


T decreasing with E

Fragmentation of hydrogen clusters

$H_3^+(H_2)_n$



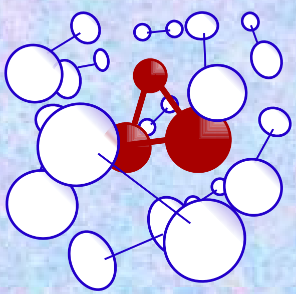


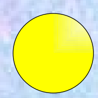
He

Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters

$H_3^+(H_2)_n$





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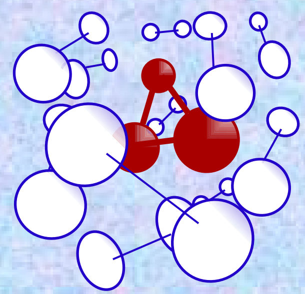

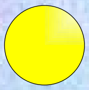

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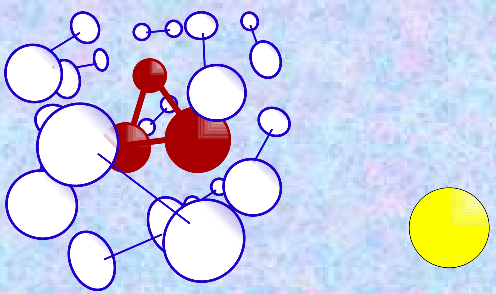

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Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters

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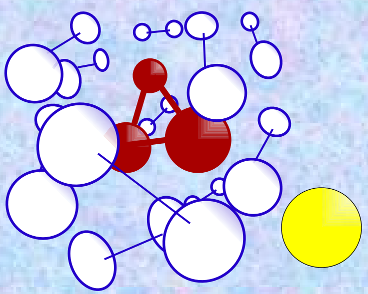
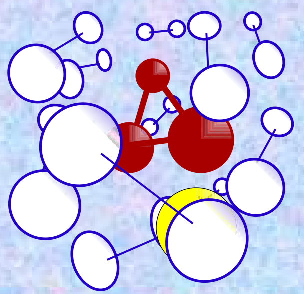


He

Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters

$H_3^+(H_2)_n$

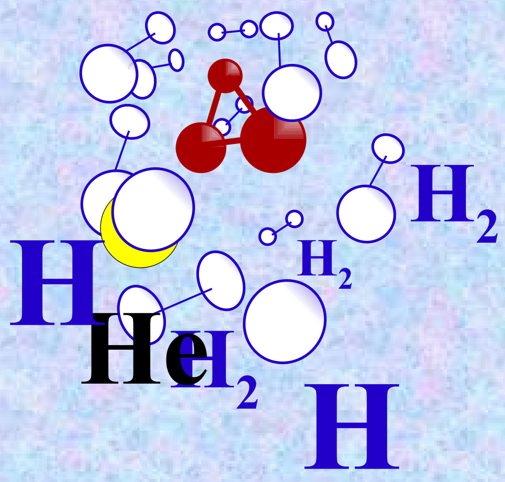


He

Gobet et al, PRL (2002)

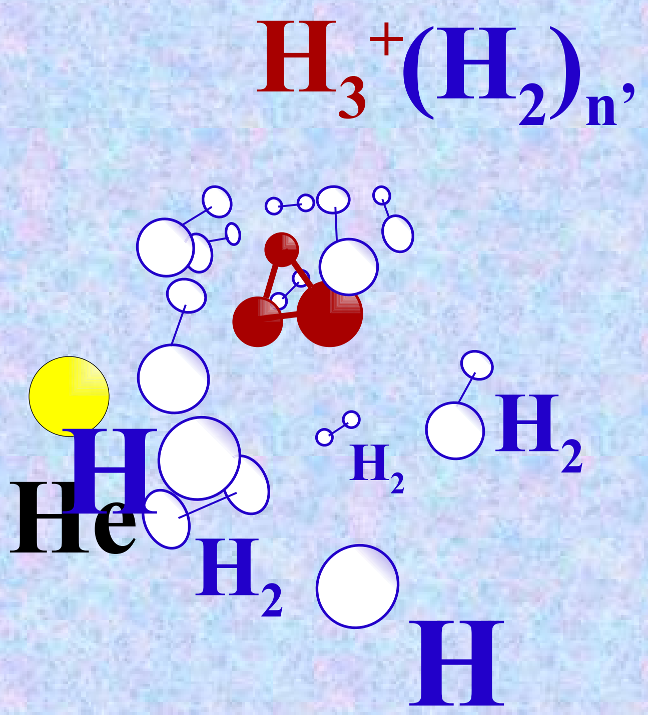
Fragmentation of hydrogen clusters

$H_3^+(H_2)_{n'}$



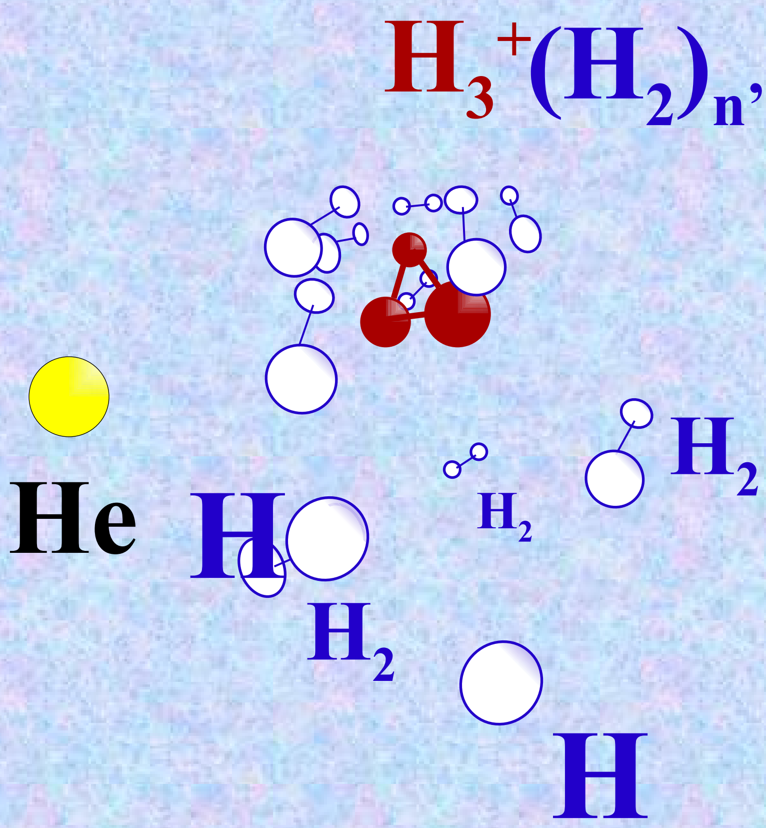
Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters



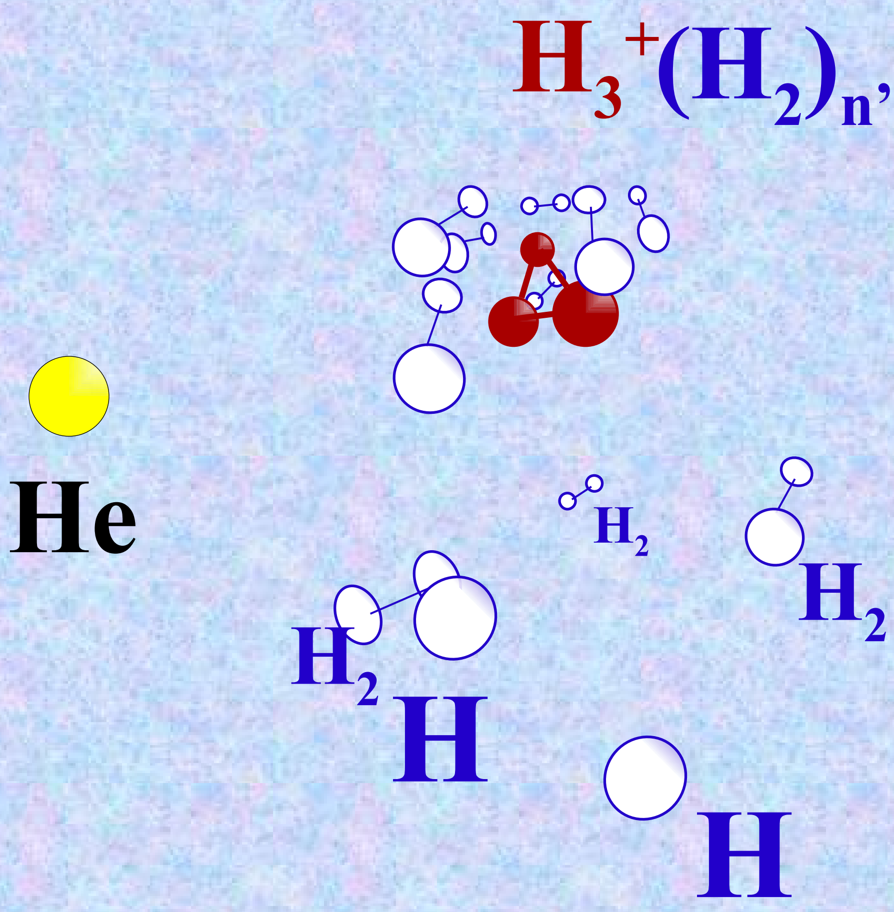
Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters



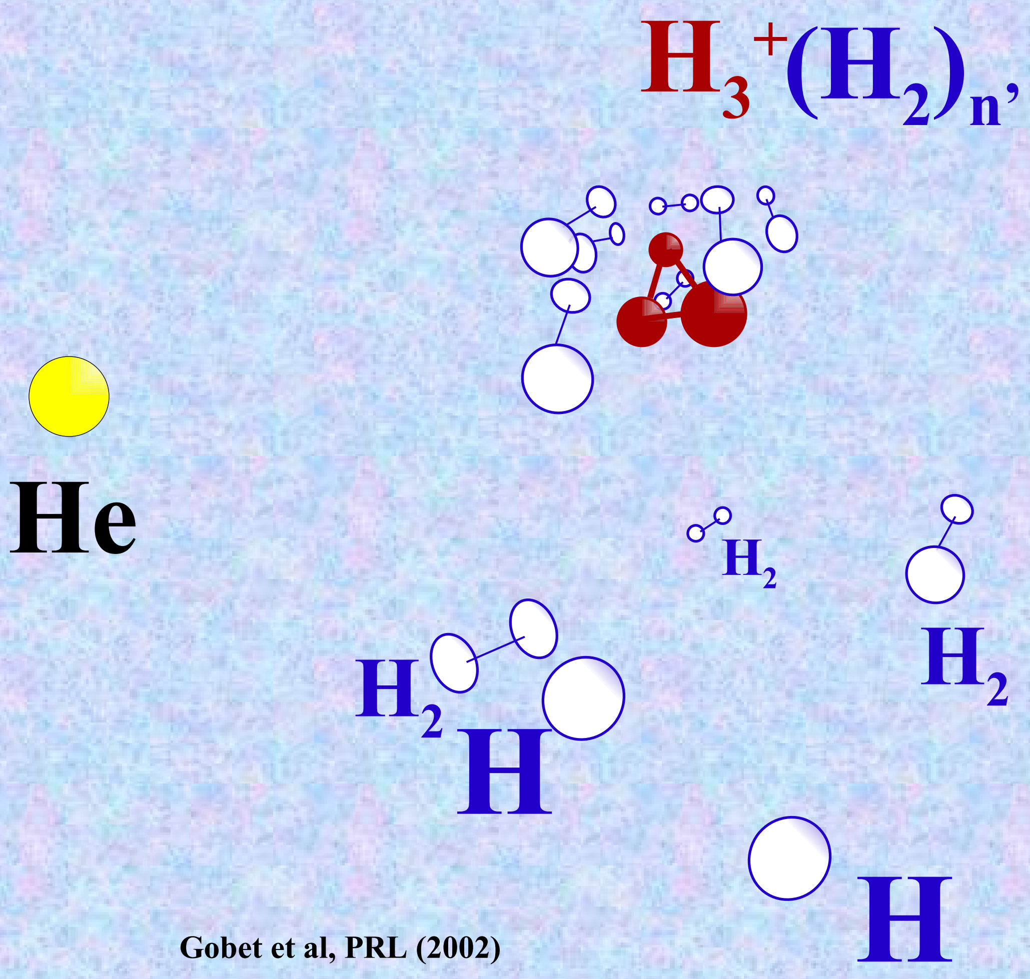
Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters



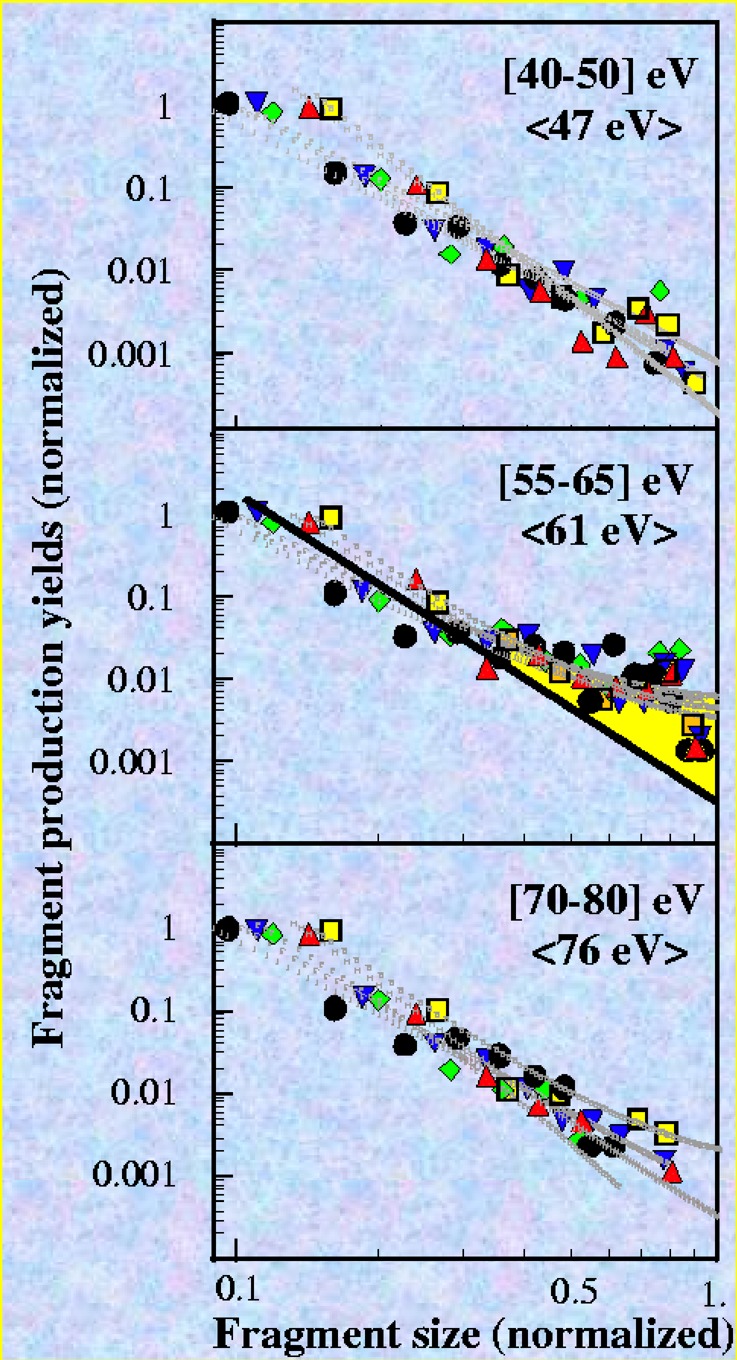
Gobet et al, PRL (2002)

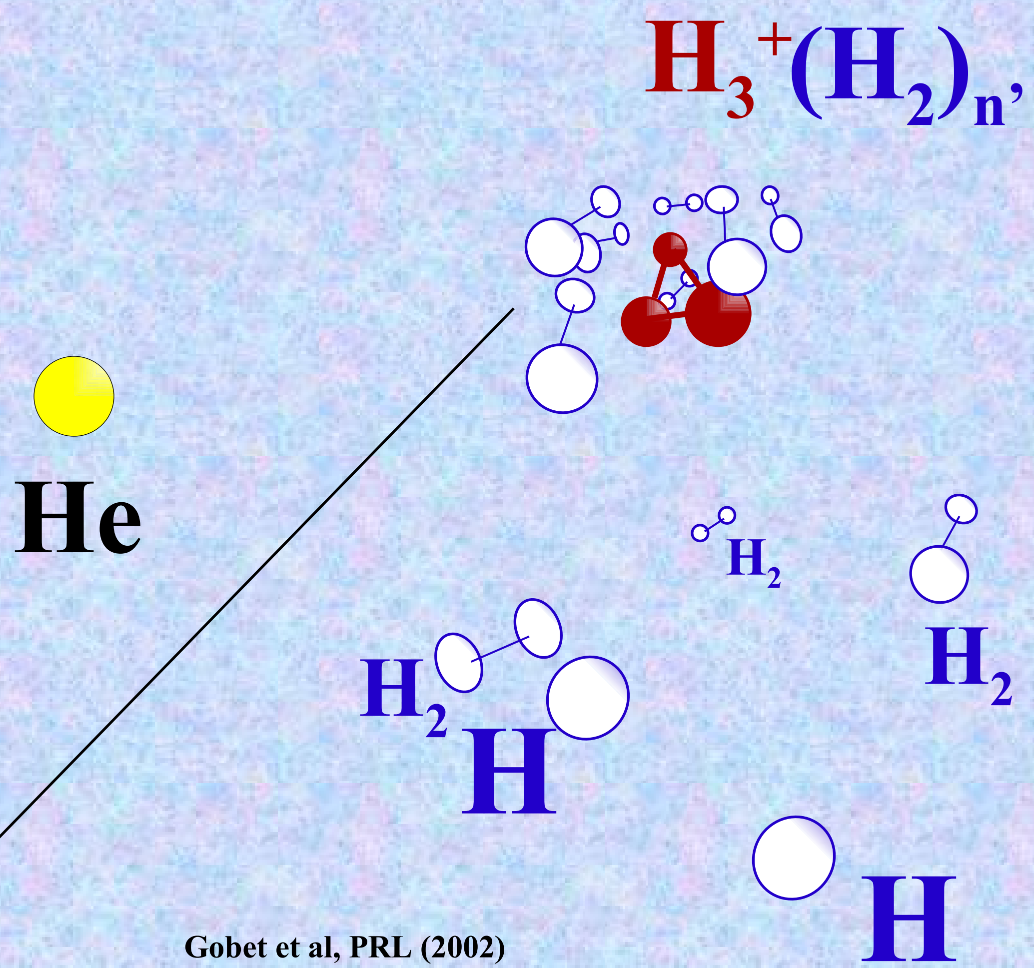
Fragmentation of hydrogen clusters



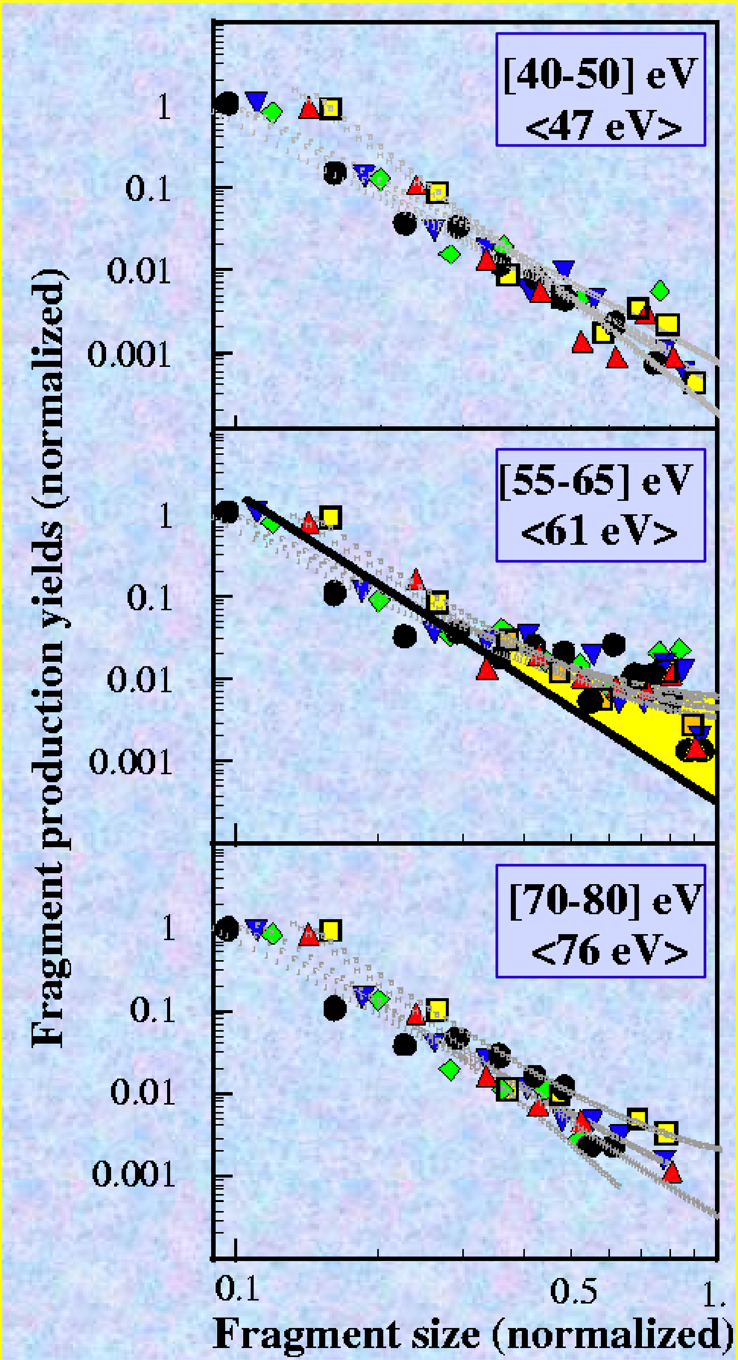
Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters





Gobet et al, PRL (2002)

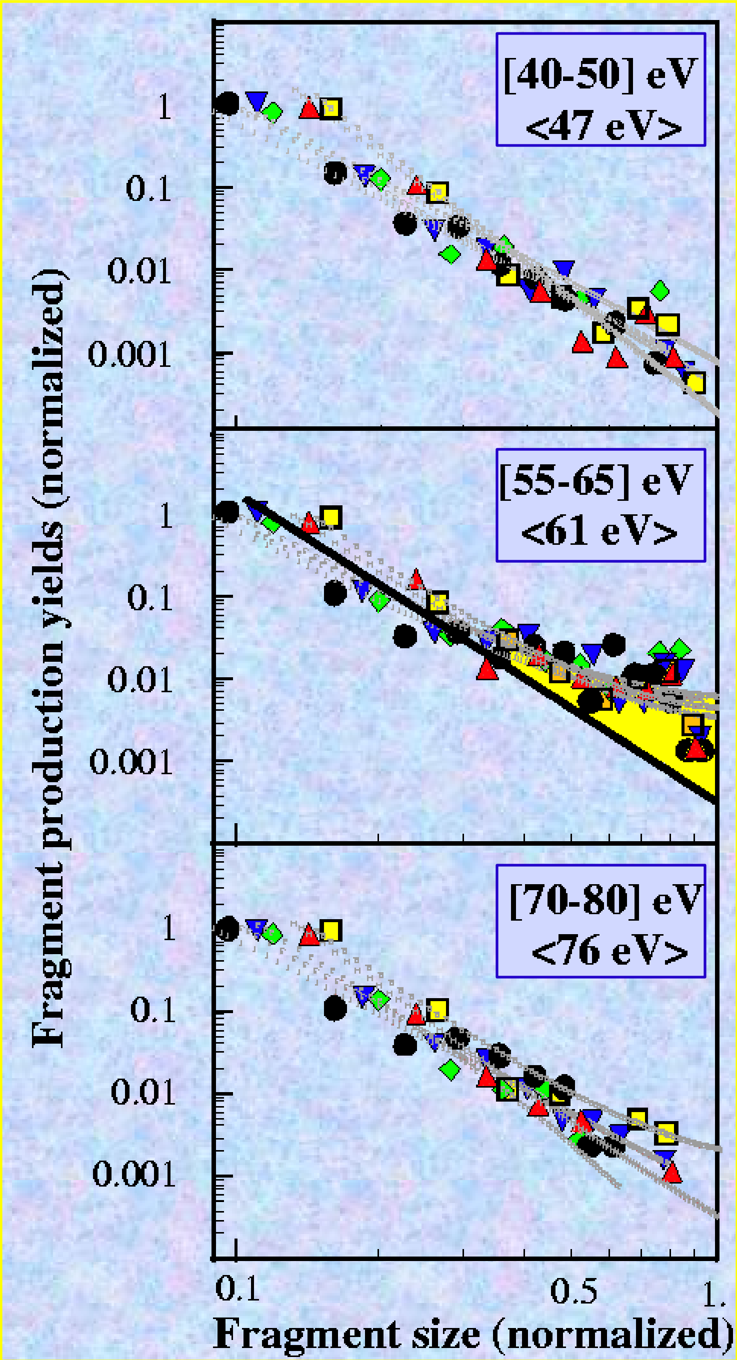


Fragmentation of hydrogen clusters

- Partition => energy

Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters

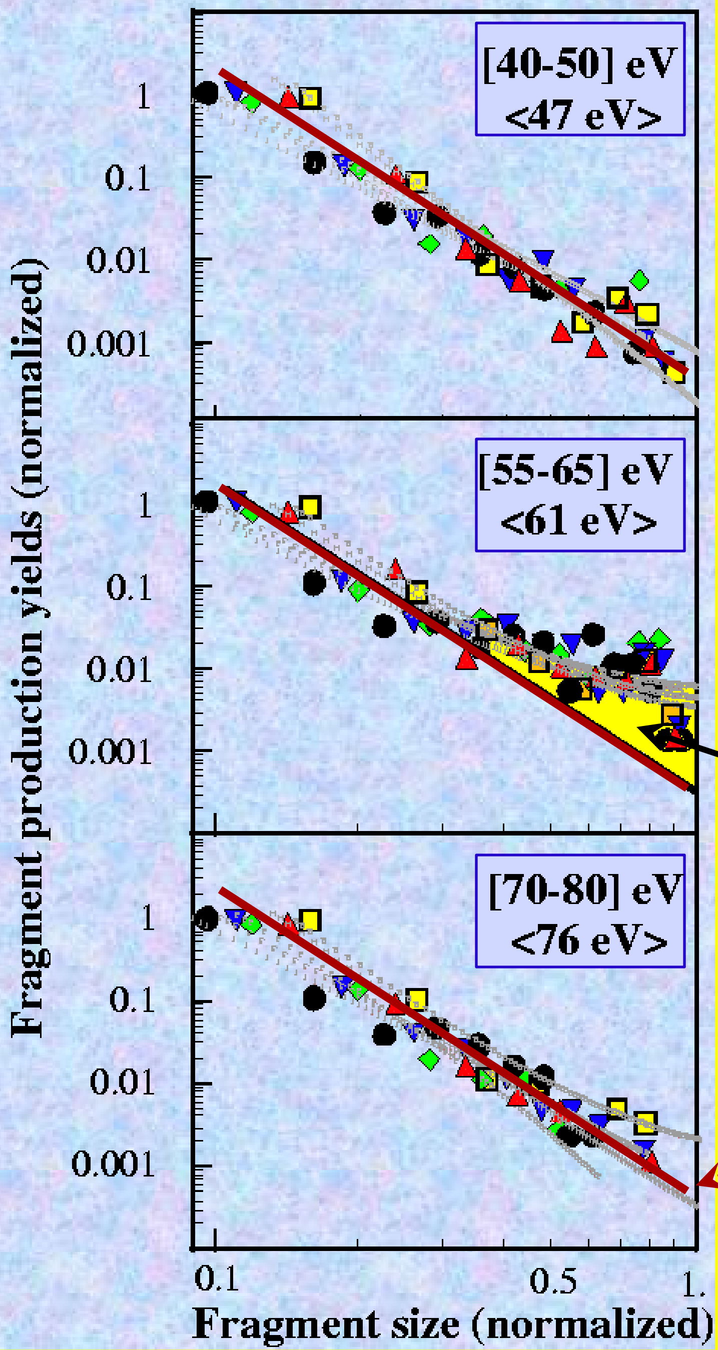


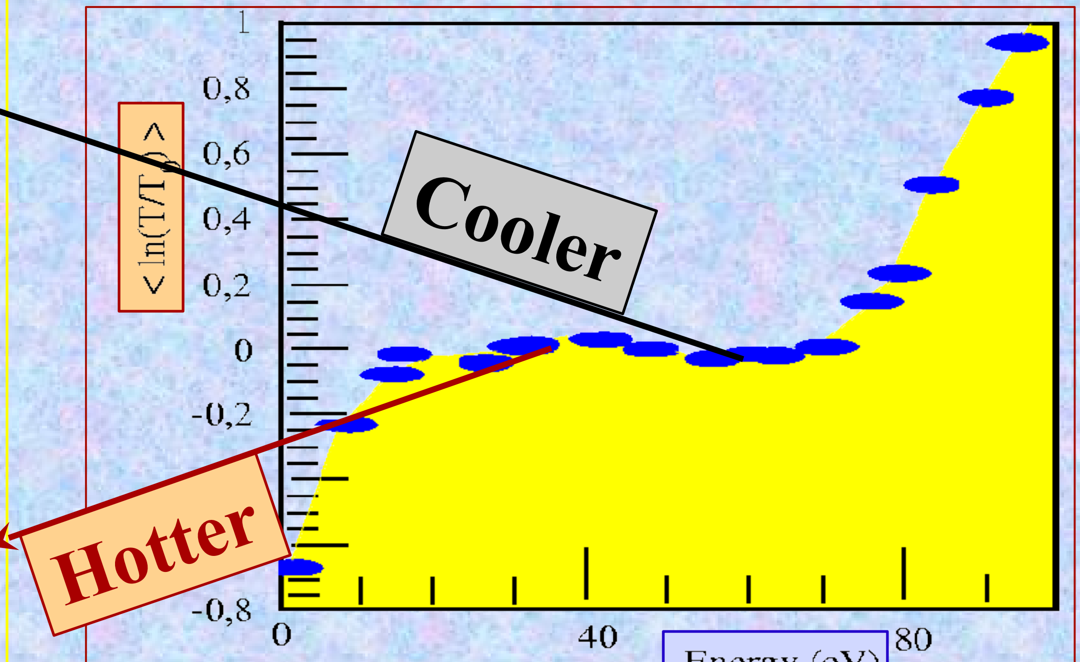
- Partition => energy
- Largest => Temperature

Gobet et al, PRL (2002)

Fragmentation of hydrogen clusters

- Partition => energy
- Largest => Temperature

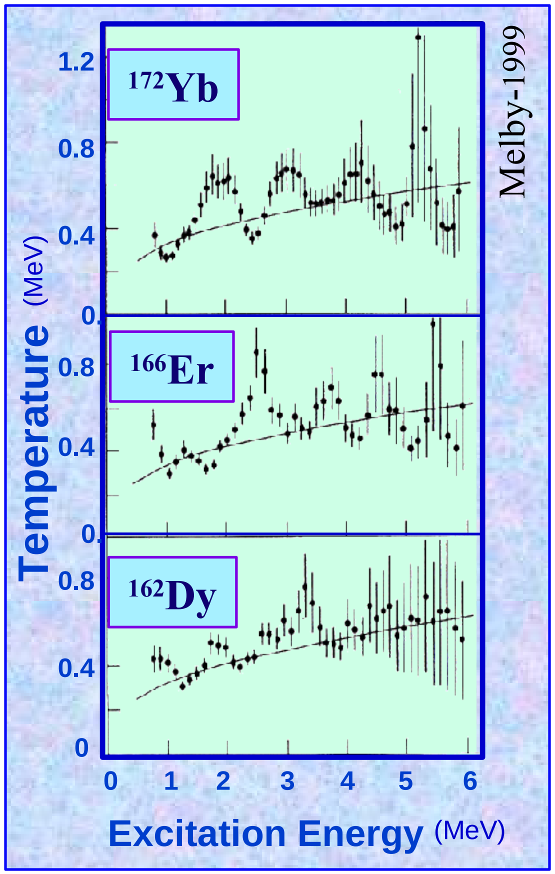




Gobet et al, PRL (2002)

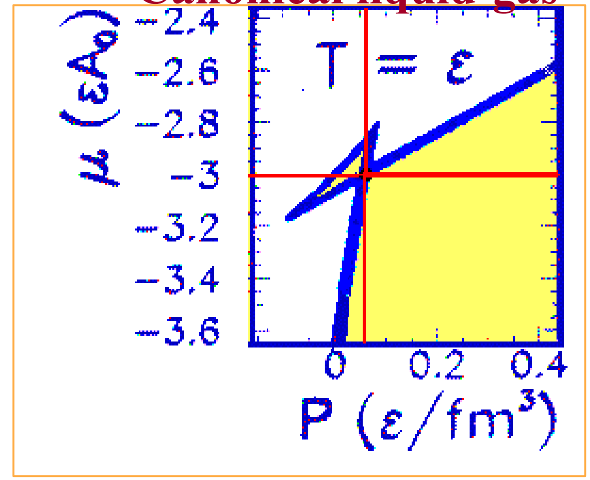
Abnormal curvatures

Nuclear superfluidity



Convex entropy

Canonical liquid-gas

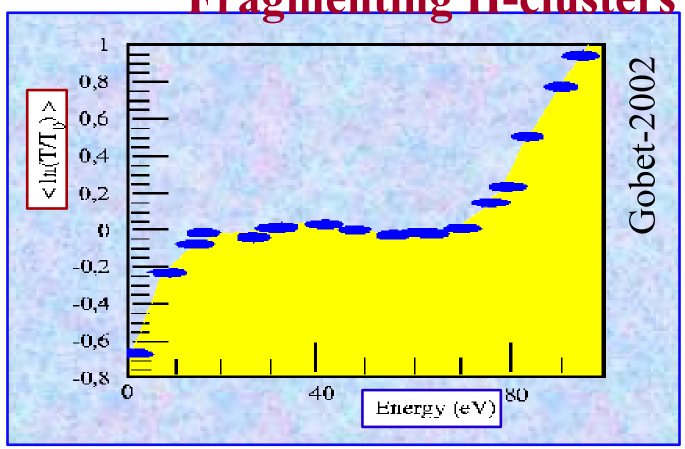


F. Gulminelli, Ph. Ch. PRL(1998)

Compressibility < 0

Susceptibility < 0

Fragmenting H-clusters



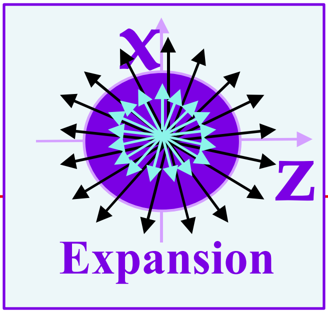
T decreasing with E

-Appendix - I -

Phase transition under flows

- Exact theory for radial flow
  - Isobar ensemble required
  - Phase transition not affected
- Example of oriented flow (transparency)
  - Exact thermodynamics
  - Effects on partitions

Radial flow at "equilibrium"



- Equilibrium = Max S under constrains
- Additional constrains: radial flow $<p_r(r)>$
  - Additional Lagrange multiplier $\gamma(r)$
  - Self similar expansion $<p_r(r)> = m\,\alpha\, r$ => $\gamma(r) = \beta\,\alpha\, r$

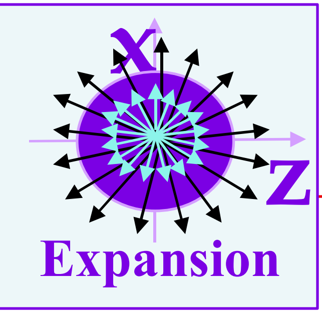
Thermal distribution in the moving frame

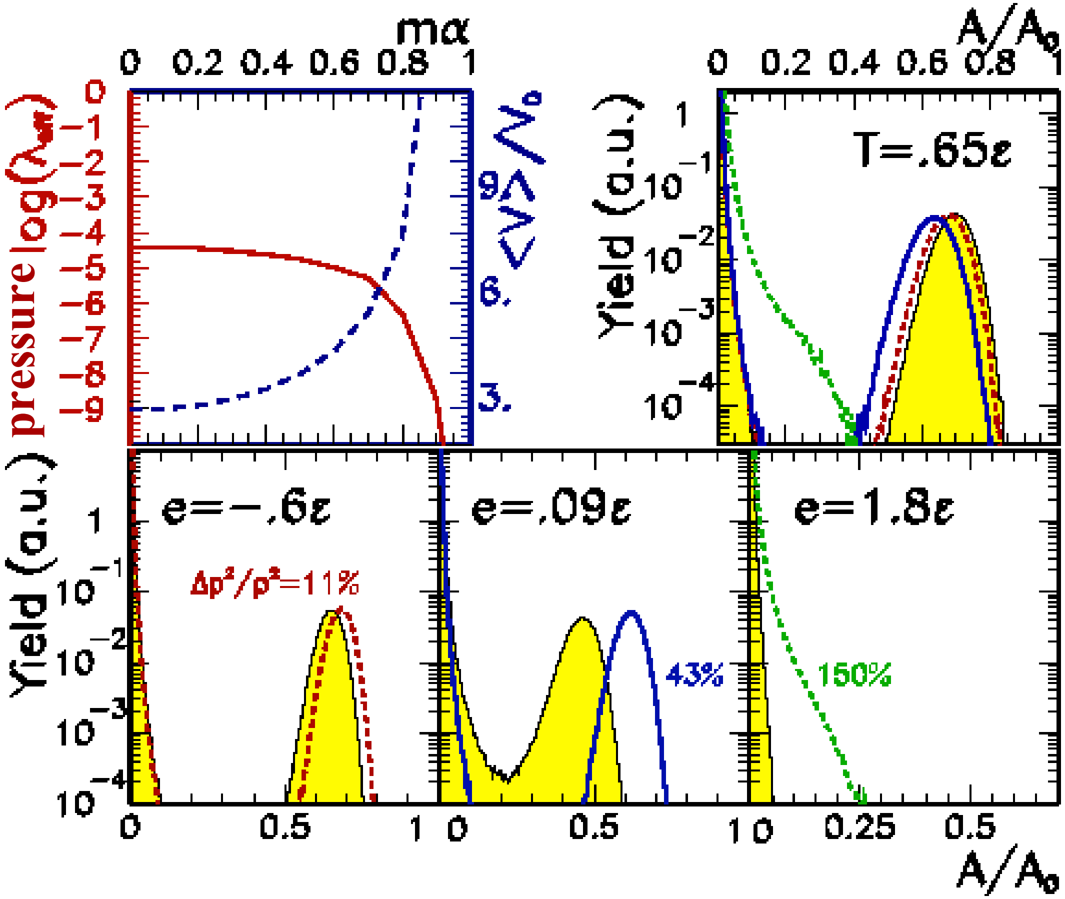
Negative pressure
Isobar ensemble

- <u>Does not affect thermo is flow subtracted</u>

Gulminelli, PC, Nucl-Th (2002)

Radial flow at "equilibrium"



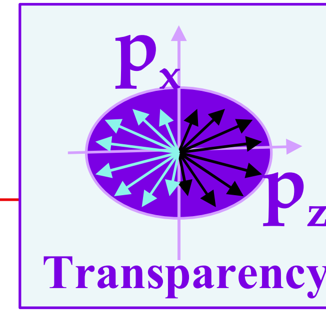


- Does no affect r partitioning
- Only reduces the pressure
- Shifts p distribution
- Changes fragment distribution: fragmentation less effective

- **Does not affect thermo is flow subtracted**

Gulminelli, PC, Nucl-Th (2002)

Transparency at "equilibrium"



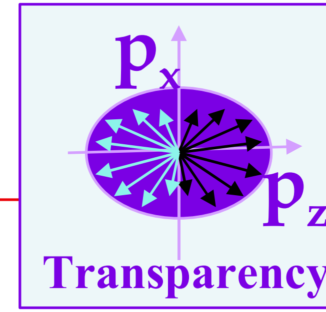
- Equilibrium = Max S under constrains
- Additional constrains: memory flow $<\tau p_z>$ { τ=-1 target, τ =1 projectile }
  - Additional Lagrange multiplier α
  - EOS leads to $<\tau p_z> = Ap_0$ with

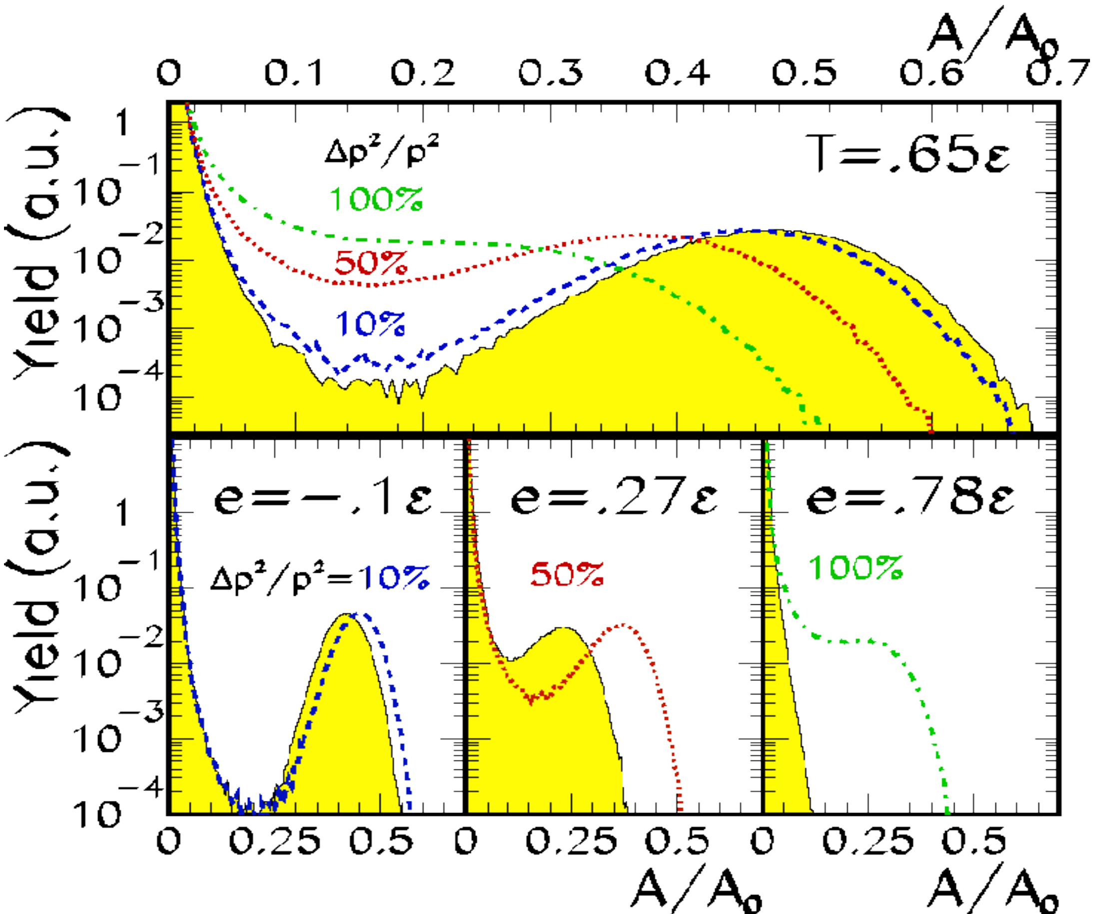
Thermal distribution
in moving frame

- Affects p space and fragments,
- Does not affect the thermo if E_{flow} subtracted

Gulminelli, PC, Nucl-Th (2002)

Transparency at "equilibrium"



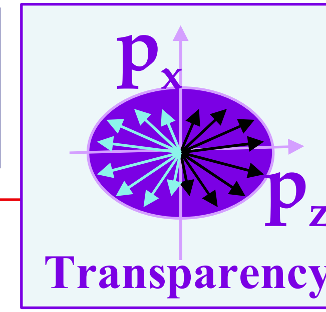


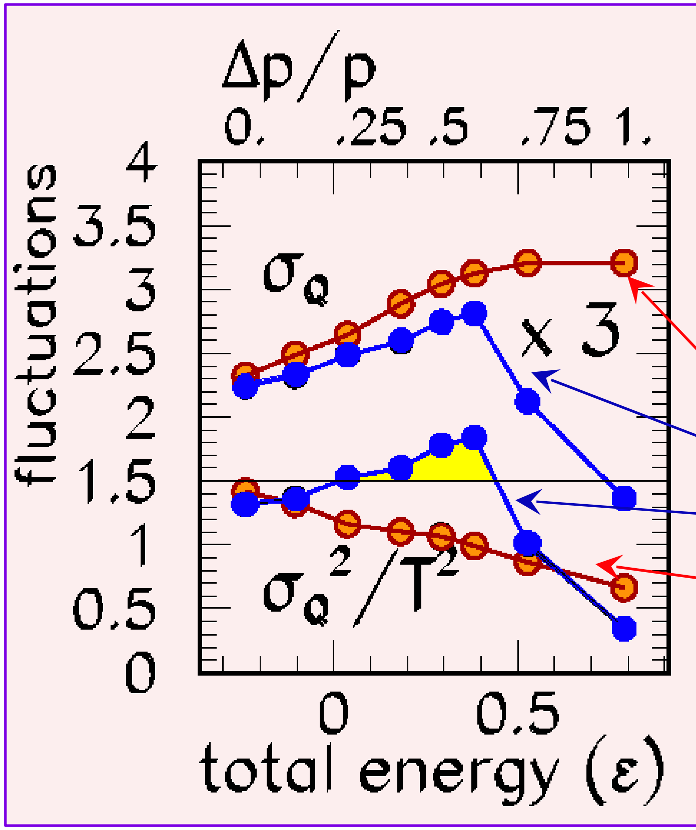
- Does no affect r partitioning
- Shifts p distribution
- Changes fragment distribution: fragmentation less effective

- **Affects p space and fragments**

Gulminelli, PC, Nucl-Th (2002)

Transparency at “equilibrium”





- Fragmentation less effective
- <u>If E_{flow} not subtracted</u>
  - Affect <E>
  - Affect T from $<E_k>$
  - Ex: Fluctuations of Q values
    - All thermal (from E = - 2ε)
    - All relative motion
- Up to 10-15% no effect

- **Affects p space and fragments**

Gulminelli, PC, Nucl-Th (2002)

- Appendix -II-

Thermo with Coulomb

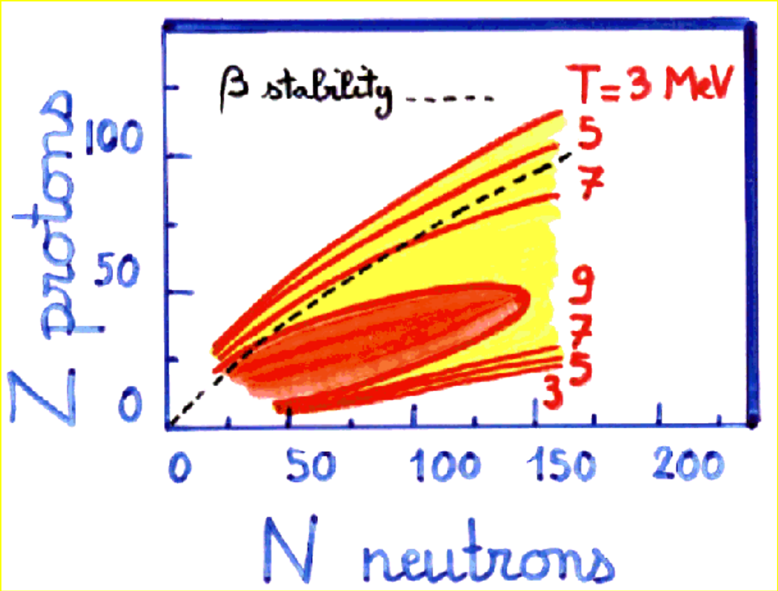
- Coulomb reduces coexistence and $C<0$ region
- Statistical treatment of Coulomb
  - (E_N,V_C) a common phase diagram for charged and uncharged systems

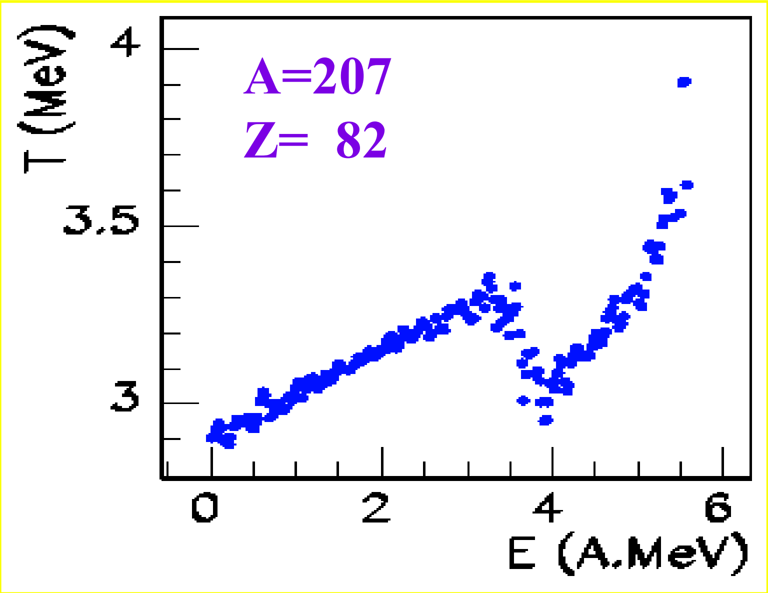
Coulomb reduces L-G transition

- **Reduces coexistence**
  - **Bonche-Levit-Vautherin**

Nucl. Phys. A427 (1984) 278

- Reduces C<0
  - Lattice-Gas
    - OK up to heavy nuclei





Gulminelli, PC, Comment to PRC66 (2002) 041601

Statistical treatment of
Non saturating forces interactions

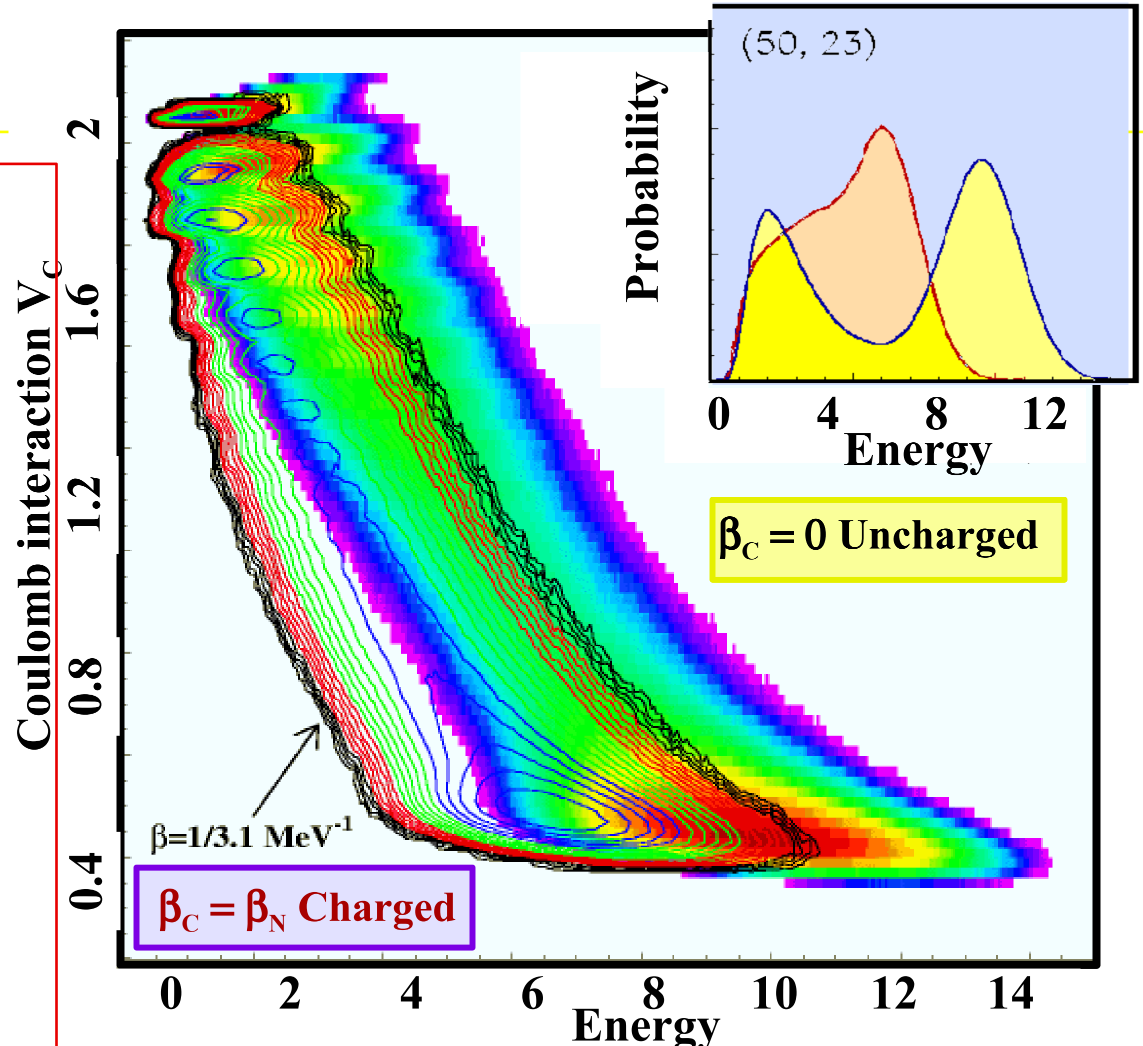
- Effective charge q = 1 charged system -
- Effective charge q = 0 uncharged system -

- **A unique framework:** $e^{-\beta E} = e^{-\beta_N E_N - \beta_C V_C}$
  - Introduce two temperatures $\beta_N = \beta$ and $\beta_C = q^2 \beta$
  - O□□□□□□□ two energies E_N and V_C
  - $\beta_C = 0$ <u>Uncharged</u> | $\beta_C = \beta_N$ <u>Charged</u>

With and without Coulomb a unique $S(E_N, V_C)$

- (E_N, V_C) a unique phase diagram
- Coulomb reduces E bimodality
  - different weight
  - rotation of E axis

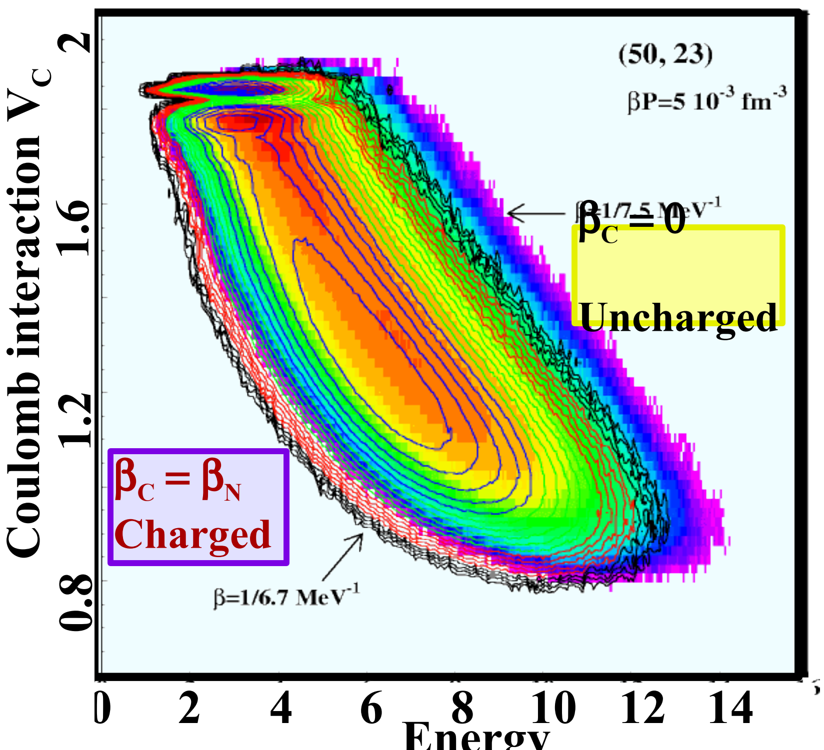
$E=E_N+V_C$, charged
$E=E_N$, uncharged

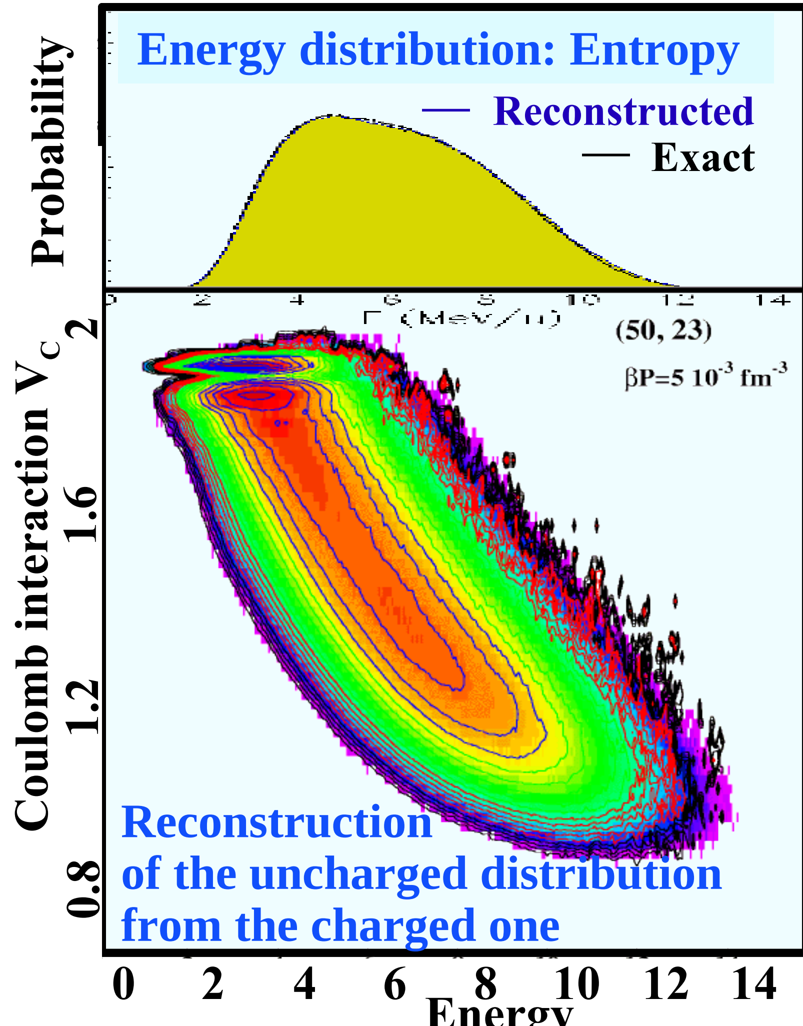


Gulminelli, PC, Raduta, Raduta, submitted to PRL

Correction of Coulomb effects

- If events overlap

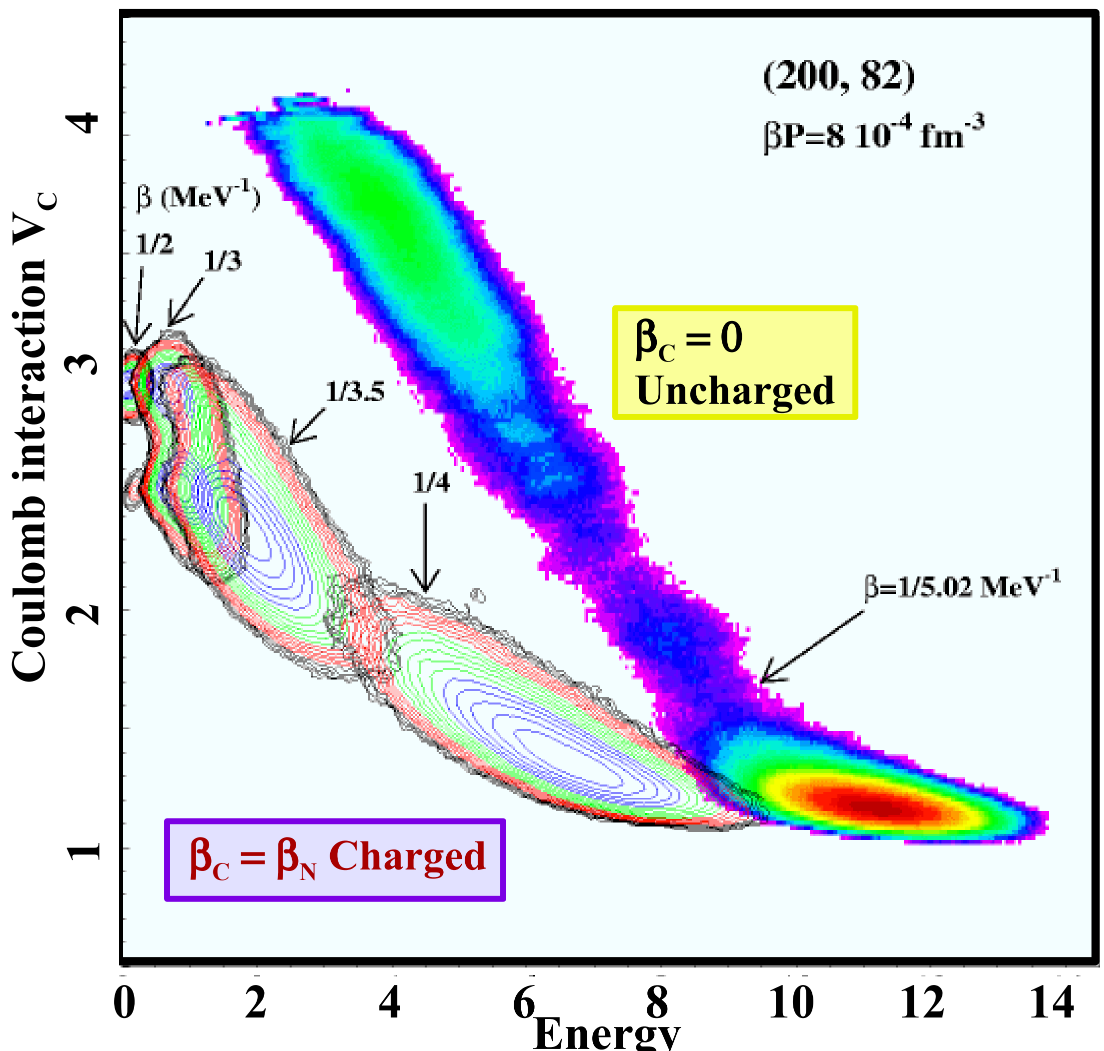




Gulminelli, PC, Raduta, Raduta, submitted to PRL

Partitions may differ for heavy nuclei

- Channels are different (cf fission)
- Re-weighting impossible
- However, a unique phase diagram (E_N V_C)



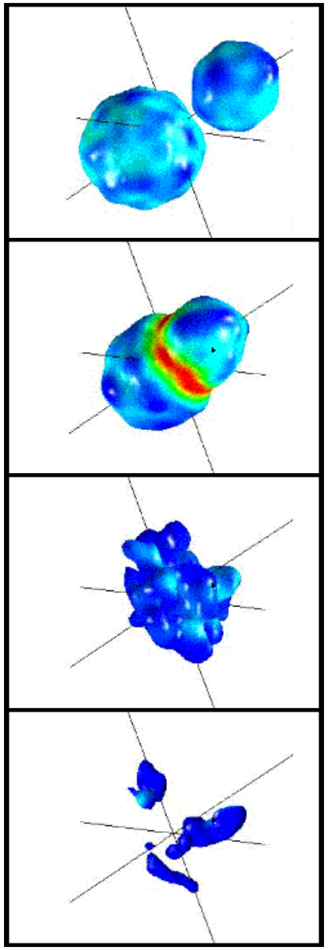
Gulminelli, PC, Raduta, Raduta, submitted to PRL

-Appendix - III -

Equilibrium in finite systems

- Macroscopic
  - One realization (event) at equilibrium
- Microscopic
  - Statistical ensemble and information
  - Ergodicity versus mixing dynamics
  - Multiplicity of equilibria
  - What is temperature?

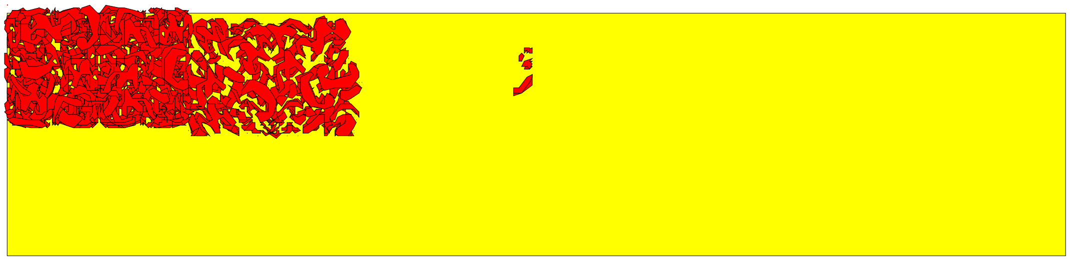
Equilibrium : What Does It Mean?



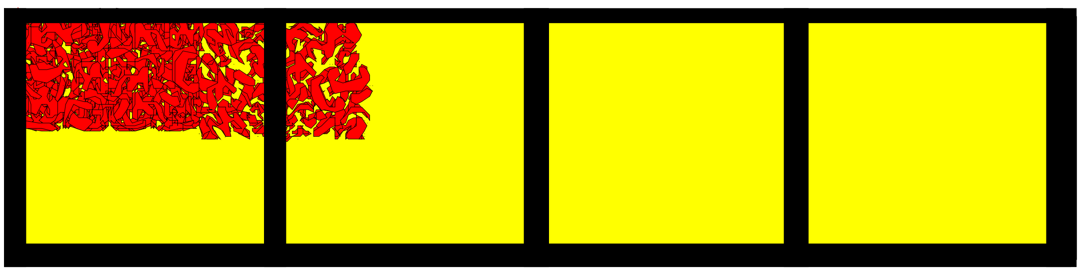
- **Finite systems**
  - In time
  - In space

- **Isolated open systems**
  - No reservoir or bath
  - No container

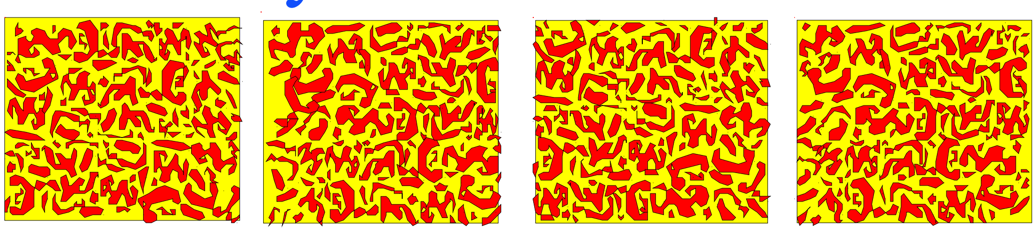
Equilibrium : What Does It Mean?



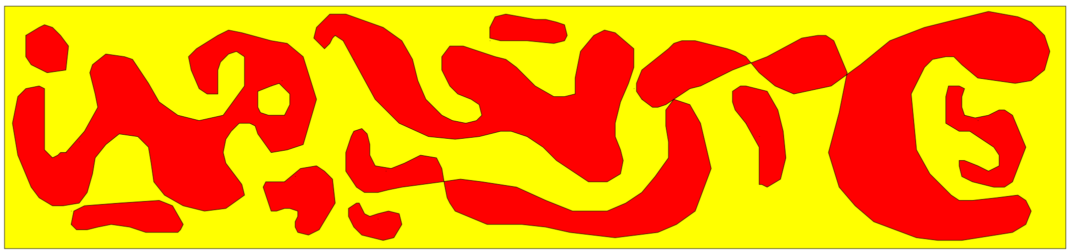
- Thermodynamics : infinite system



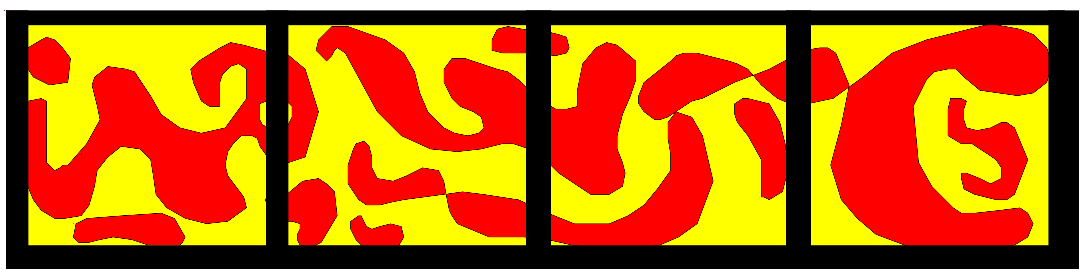
- One ∞ system = ensemble of ∞ sub-systems



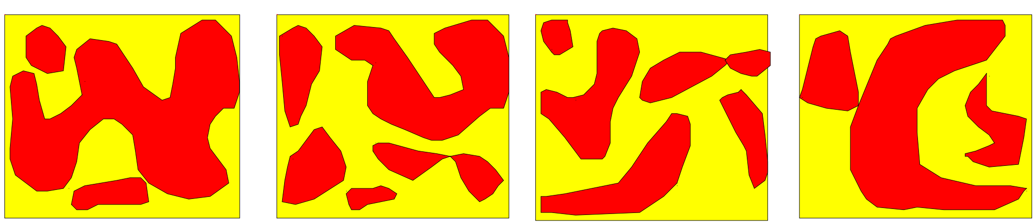
Equilibrium : What Does It Mean?



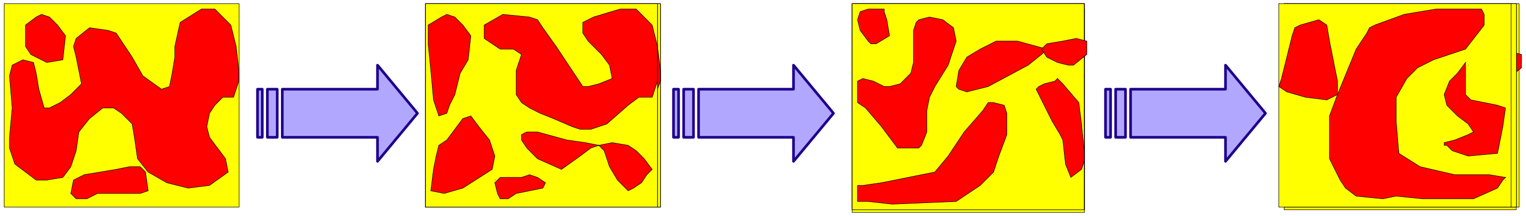
- Finite system



- Cannot be cut in sub-systems

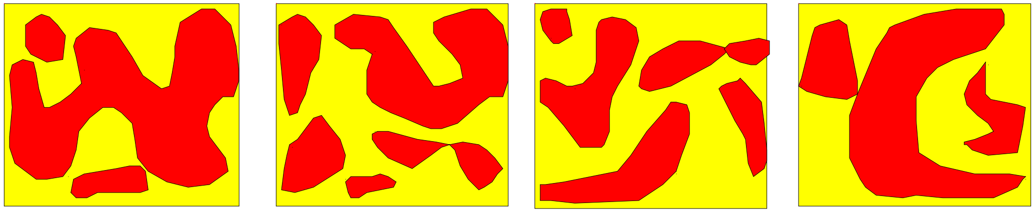


Equilibrium : What Does It Mean?

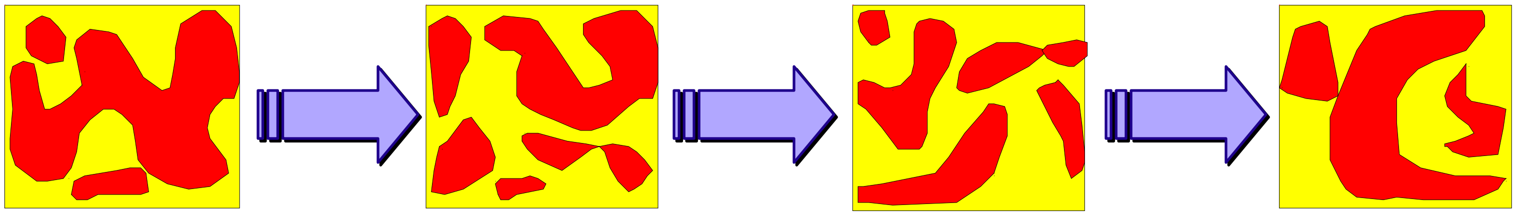


- One small system in time   if it is ergodic
  Is a statistical ensemble

- Cannot be cut in sub-systems



Equilibrium : What Does It Mean?



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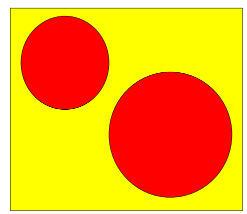
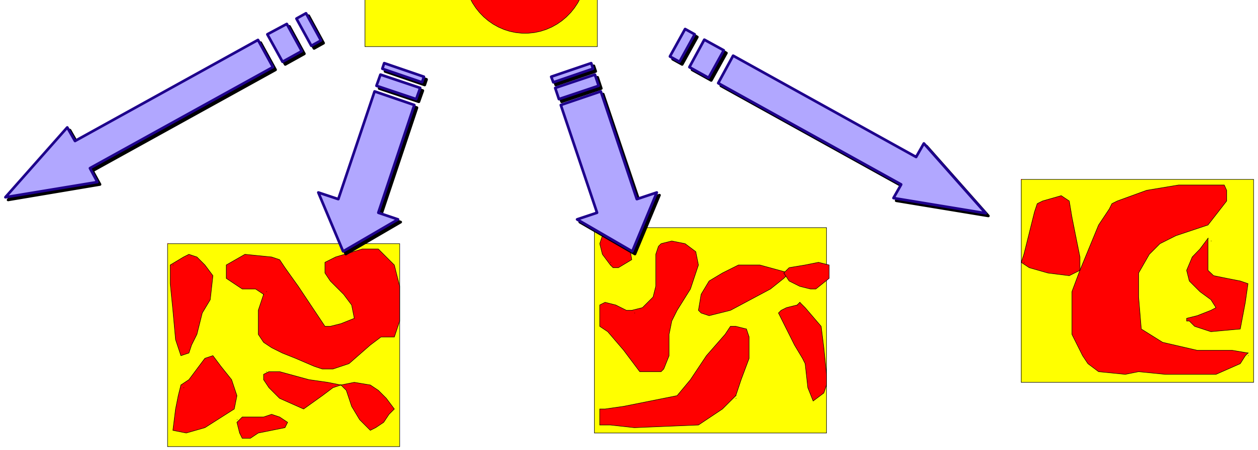
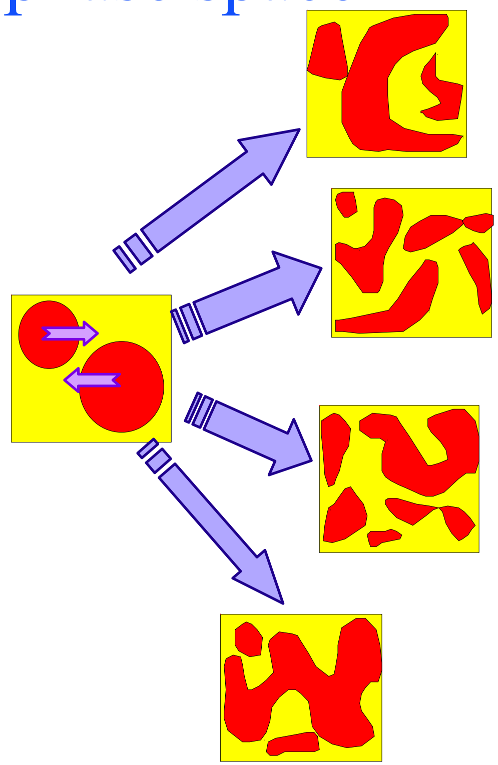


- Many events   if it is chaotic



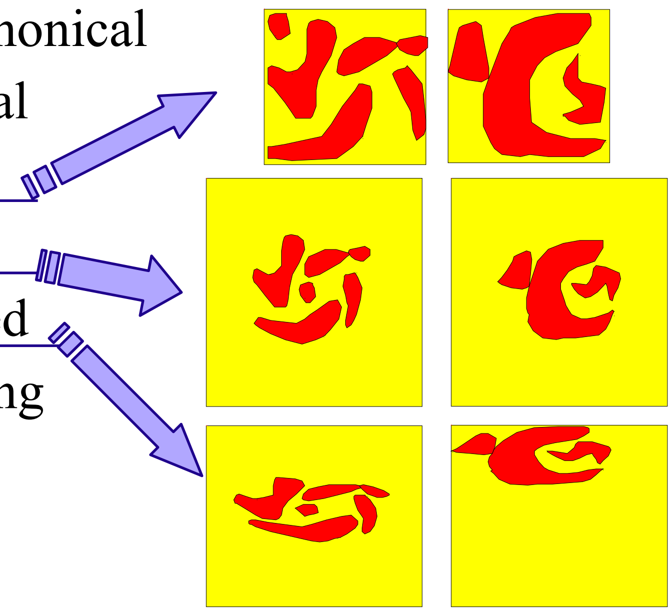
Statistics from minimum information

- "Chaos" populates phase space



- Dynamics: global variables
- Statistics: population
  - ✦ Many different ensembles

E	Microcanonical	
$\langle E \rangle$	Canonical	
V	Isochore	
$\langle r^3 \rangle$	Isobare	
$\langle Q_2 \rangle$	Deformed	
$\langle p.r \rangle$	Expanding	
$\langle A \rangle$	Grand	
$\langle L \rangle$	Rotating	
…	Others	



What is temperature ?

L. Boltzmann

- W equiprobable microstates

  Entropy = disorder (-information)

- T is the entropy increase

R. Clausius

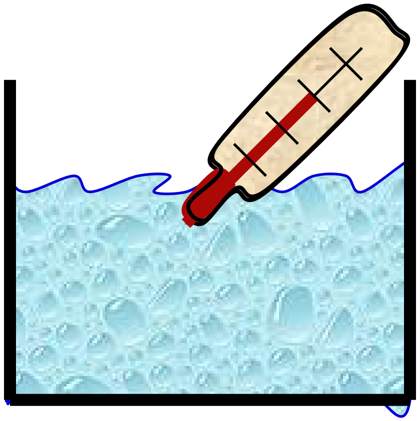
What is temperature ?

- The microcanonical temperature

$$S = k \log W \qquad\qquad \delta S = T^{-1}\,\delta E$$

What is temperature ?

- **It is what thermometers measure.**



$$E = E_{thermometer} + E_{system}$$

- The microcanonical temperature

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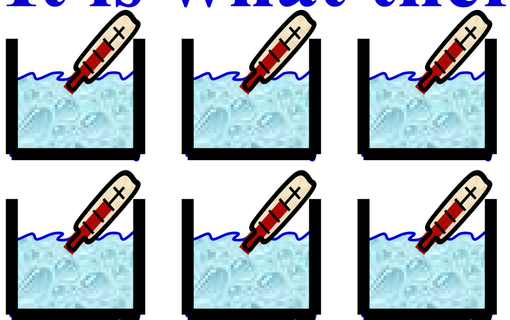
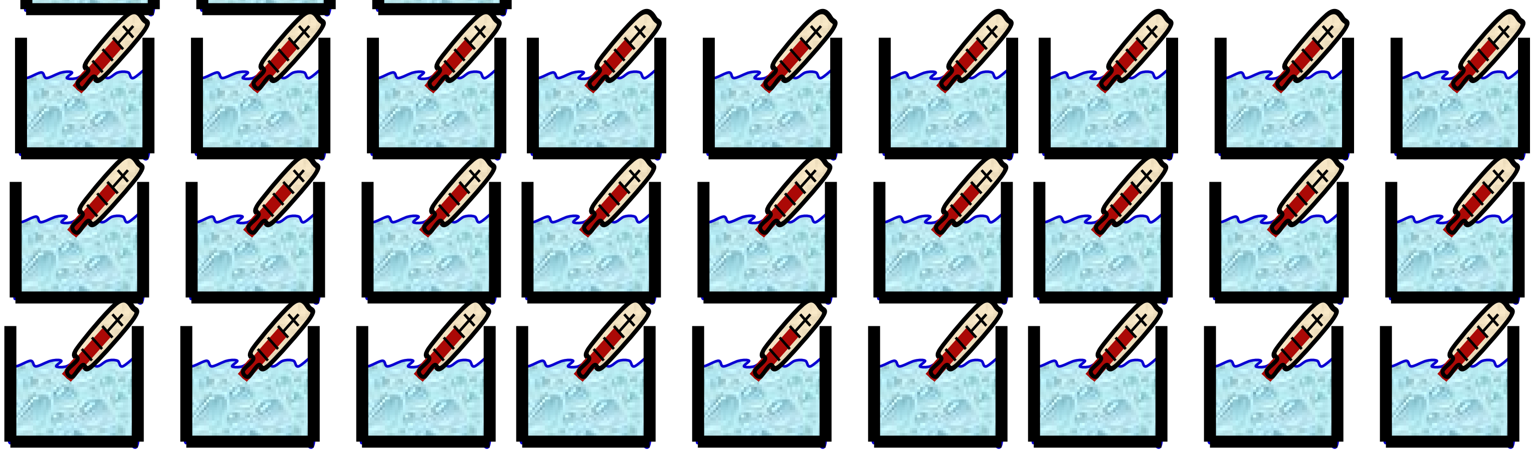


$$E = E_{thermometer} + E_{system}$$

Distribution of microstates



- The microcanonical temperature

$$S = k \log W \qquad \delta S = T^{-1} \delta E$$

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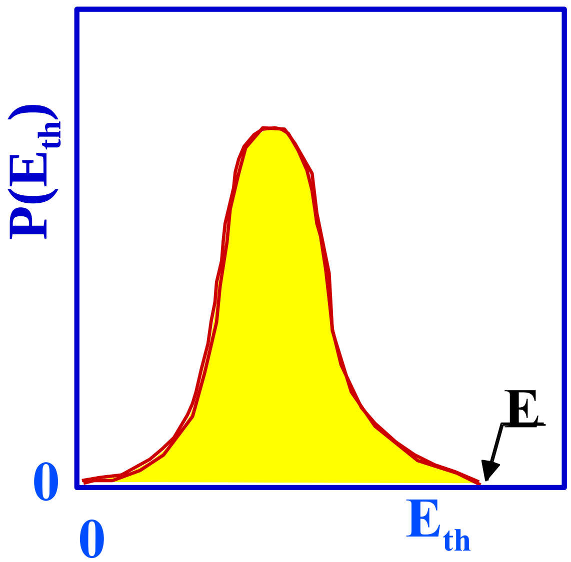


$$\mathbf{E = E_{th} + E_{sys}}$$

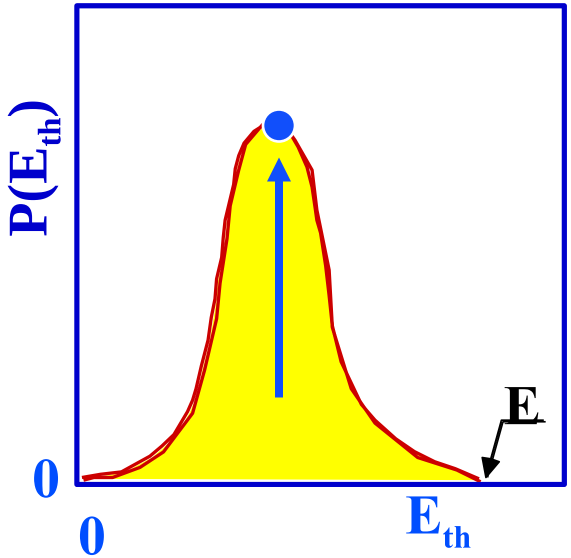
Distribution of microstates

- The microcanonical temperature

$$S = k \log W \qquad \delta S = T^{-1}\,\delta E$$

What is temperature ?

- **It is what thermometers measure.**



$$E = E_{th} + E_{sys}$$

Equiprobable microstates

$$P(E_{th}) \propto W_{th}(E_{th}) * W_{sys}(E-E_{th})$$

$$\max P \Rightarrow \delta \log W_{th} - \delta \log W_{sys} = 0$$

$$\max P \Rightarrow \delta S_{th} - \delta S_{sys} = 0$$

Most probable partition: $T_{th} = T_{sys}$

- The microcanonical temperature

$$S = k \log W$$

$$\delta S = T^{-1} \delta E$$

-Appendix -IV-

C<0 in Liquid gas transition

- Negative heat capacity in fluctuating volume ensemble
  - Difference between C_P and C_V
- Difference between C and E(T)
  - Role of the thermo transformation
- Negative heat capacity in statistical models and Channel opening

- a -

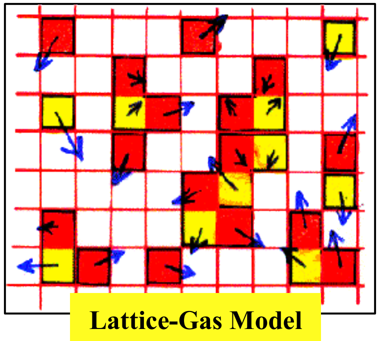
Role of volume fluctuations

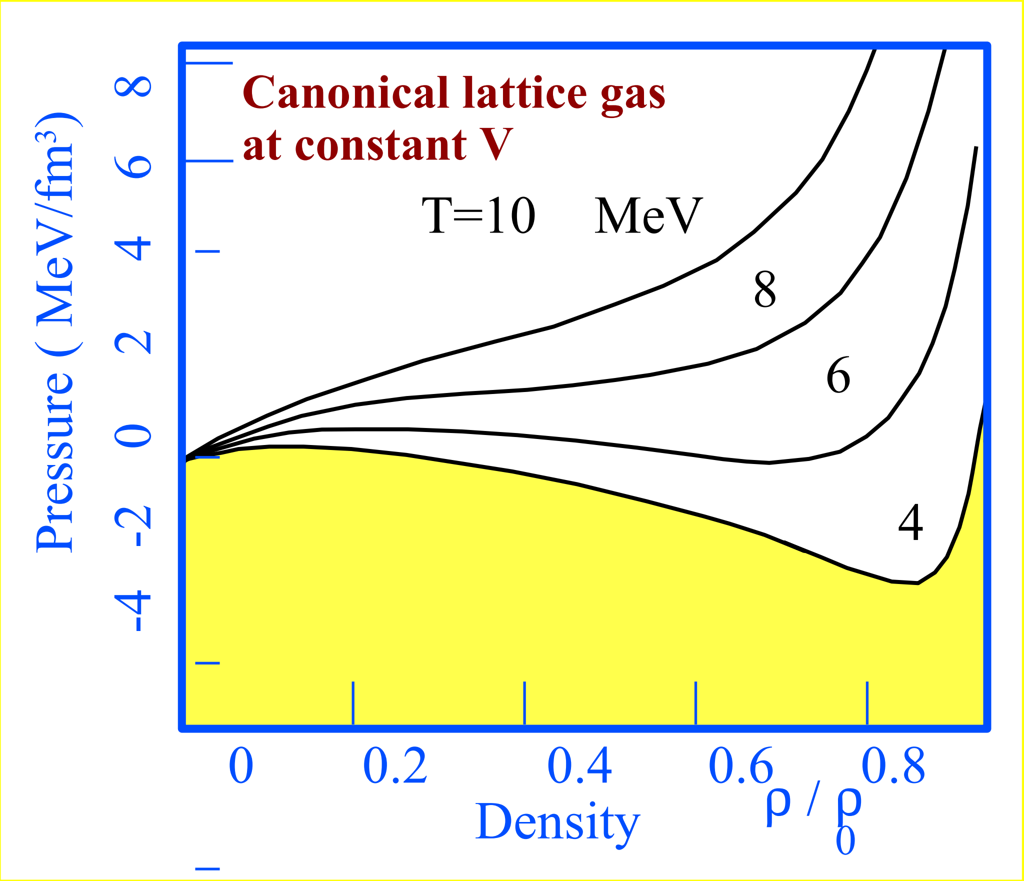
Fixed volume Cv>0 but K<0

Open systems Cp<0

Volume: order parameter

L-G in a box: V=cst



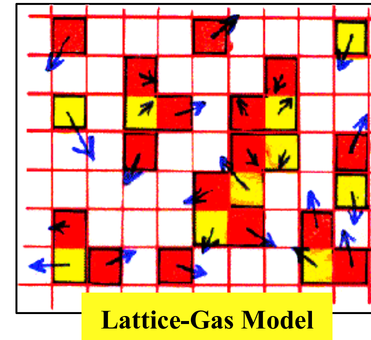


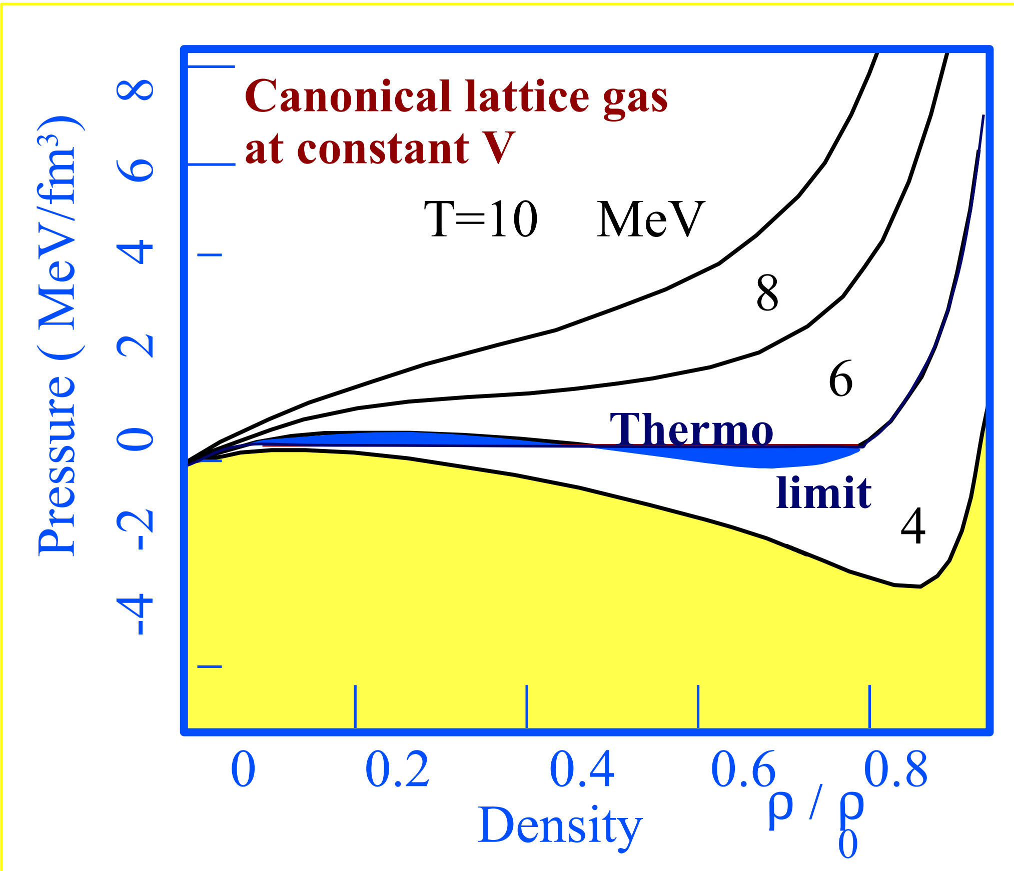
- Negative compressibility

Gulminelli & PC PRL 82(1999)1402

Volume: order parameter

L-G in a box: V=cst

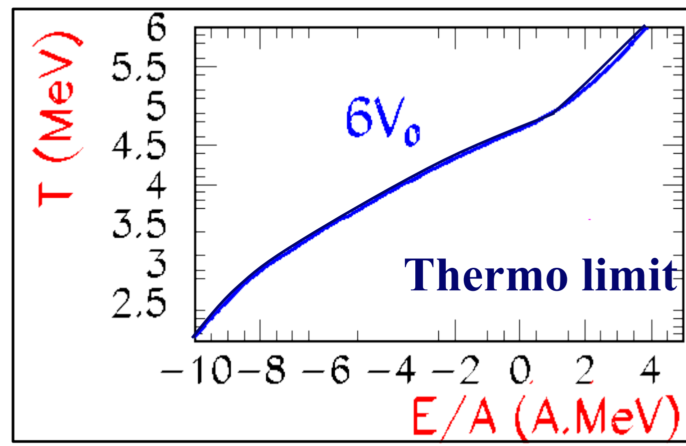




- Negative compressibility

  Gulminelli & PC PRL 82(1999)1402

- $C_V > 0$



See specific discussion for large systems

Pleimling and Hueller, J.Stat.Phys.104 (2001) 971

Binder, Physica A 319 (2002) 99.

Gulminelli et al., cond-mat/0302177, Phys. Rev. E.

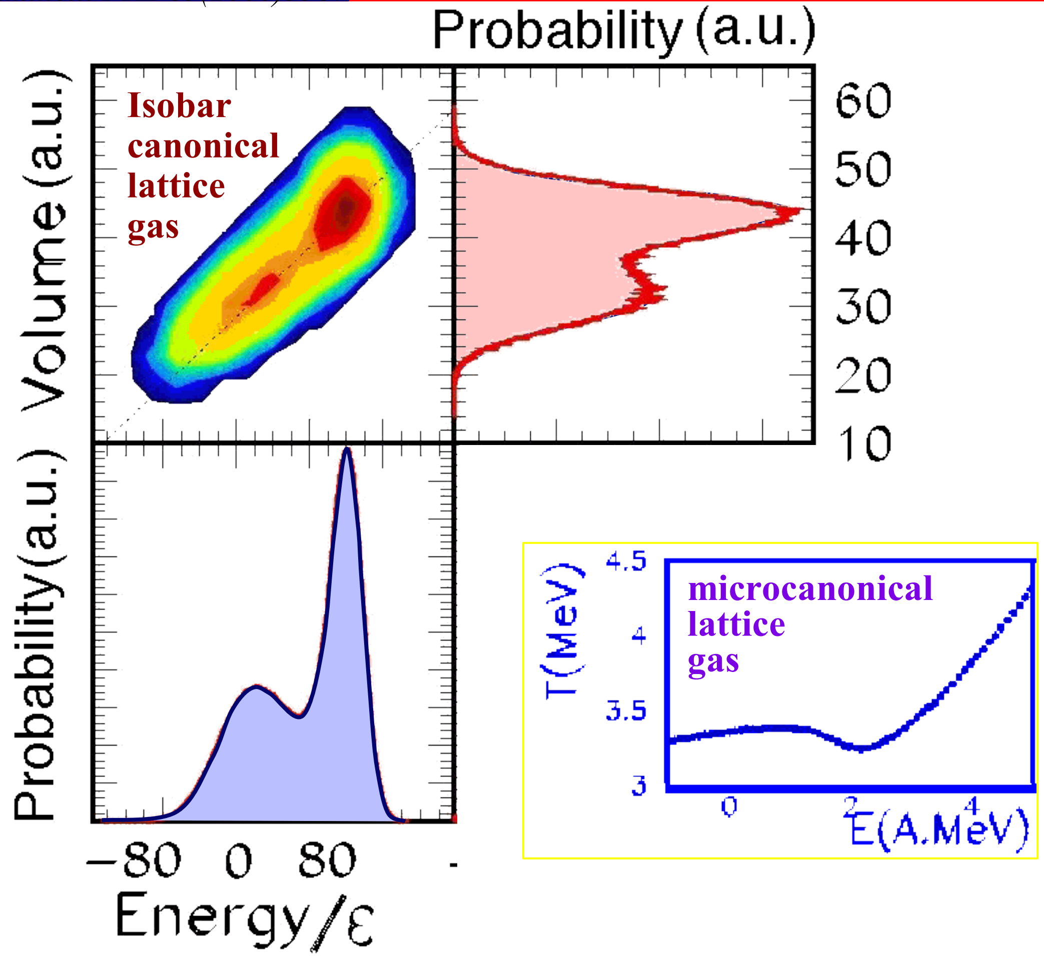
Open systems (no box)
Fluctuating volume

PC, Duflot & Gulminelli PRL 85(2000)3587

Isobar ensemble

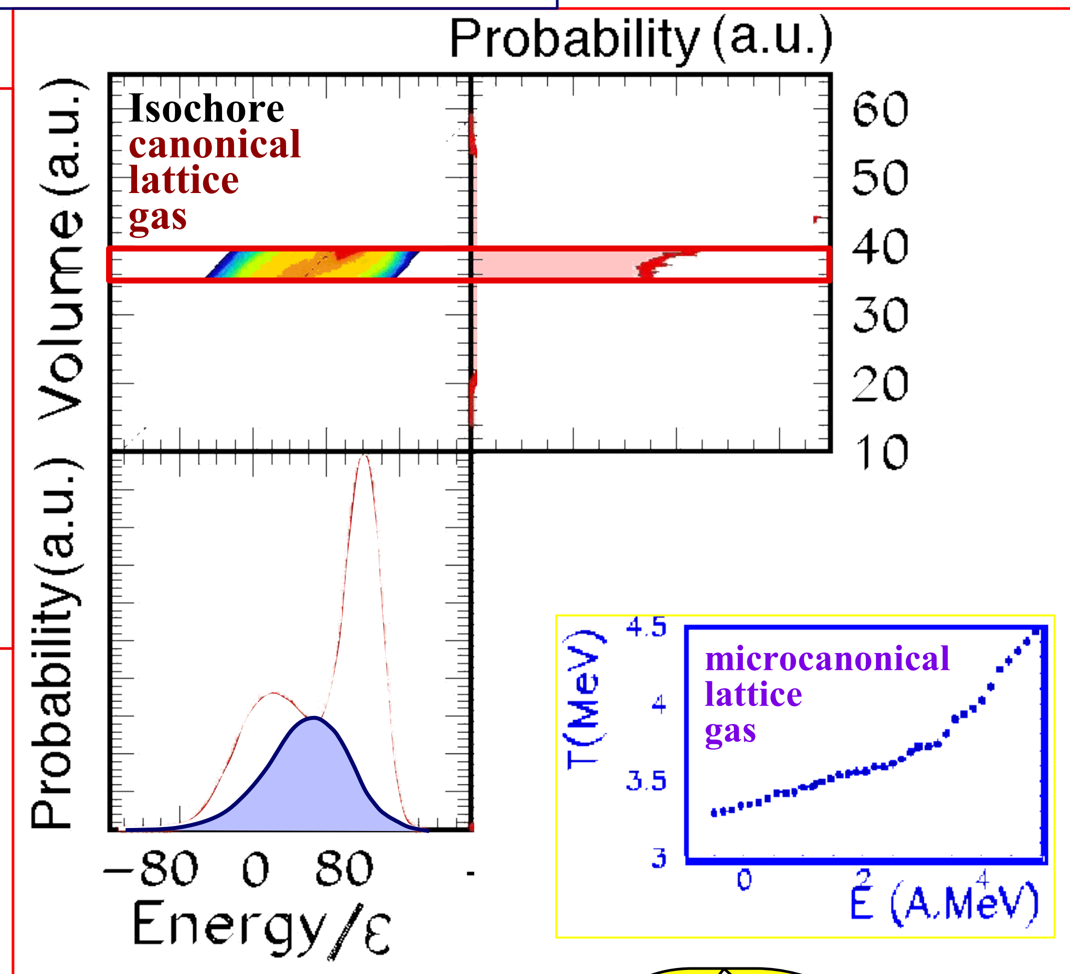
$P^{(i)} \propto \exp{-\lambda V^{(i)}}$

- **Bimodal P(V)**
  - **Negative compressibility**
- **Bimodal P(E)**
  - **Negative heat capacity C_p<0**



Constrain on V
i.e. on order paramameter

- **Suppresses bimodal P(E)**
  - **No negative heat capacity $C_V > 0$**



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- b -

Pressure dependent EOS

Caloric Curves not EOS

Fluctuations measure EOS

Volume dependent T(E)

- 2-D EOS

E ⟶ T

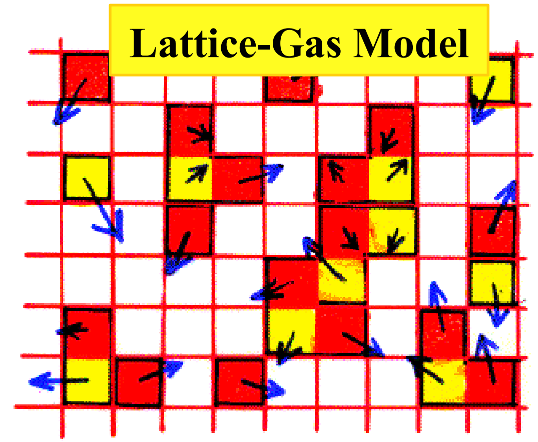
V ⟶ P

Temperature

Energy

Pressure

Volume dependent T(E)



- **2-D EOS**

E ⟶ T

V ⟶ P

Temperature

Energy

Pressure

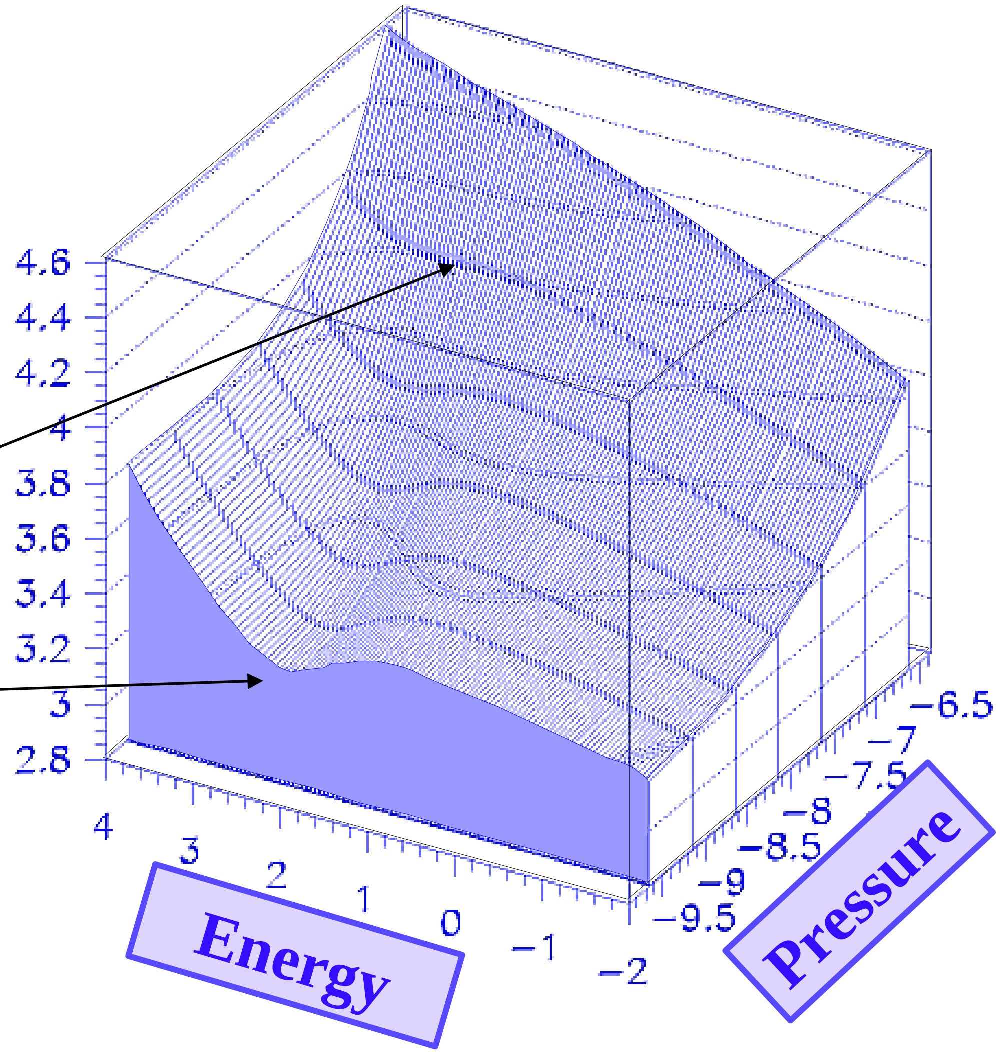
Volume dependent T(E)

- **2-D EOS**

E ⟶ T

V ⟶ P

  - **Critical point**
  - **First order Liquid gas Phase Transition**



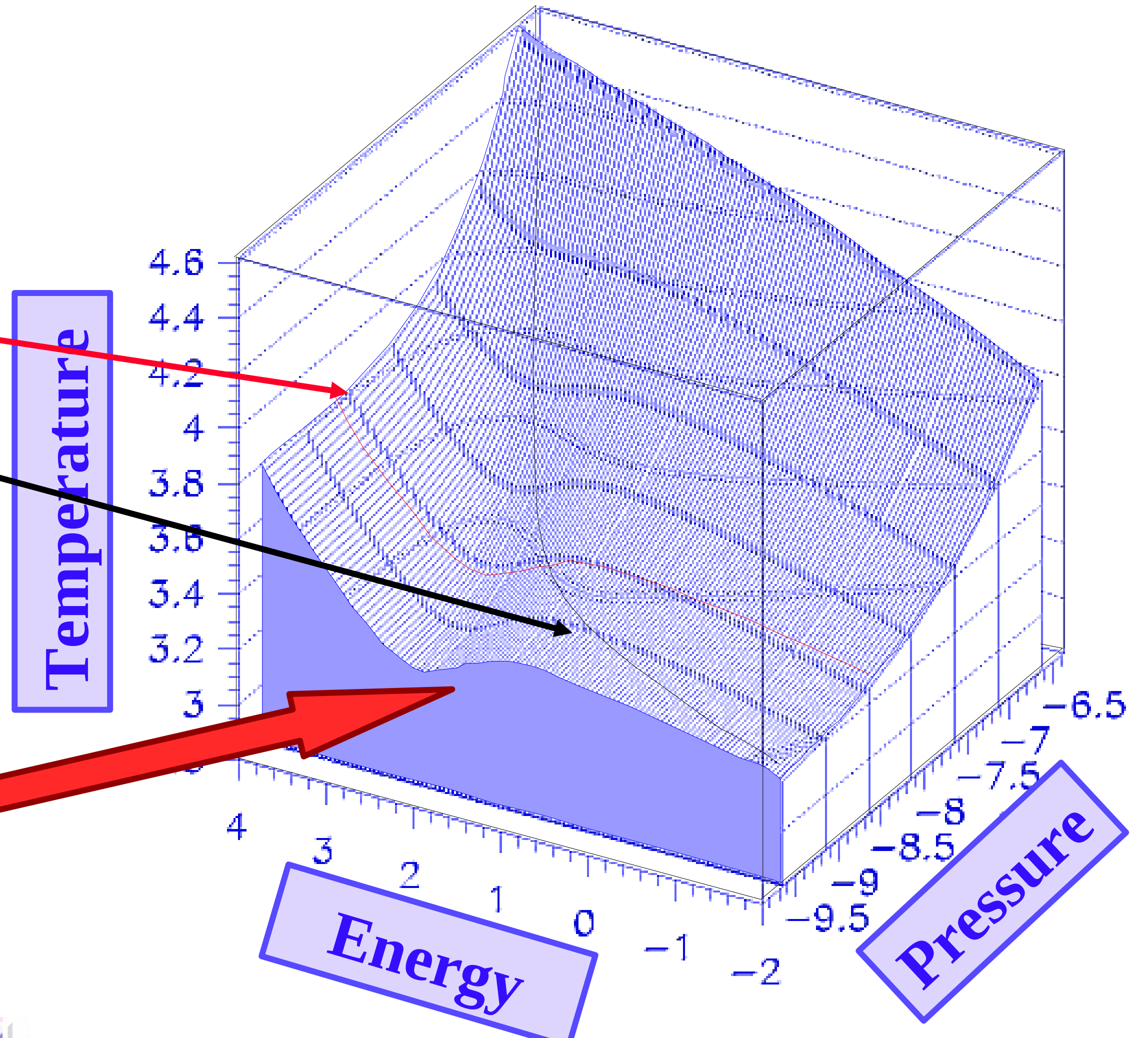
Volume dependent T(E)

- **Transformation dependent Caloric Curve**

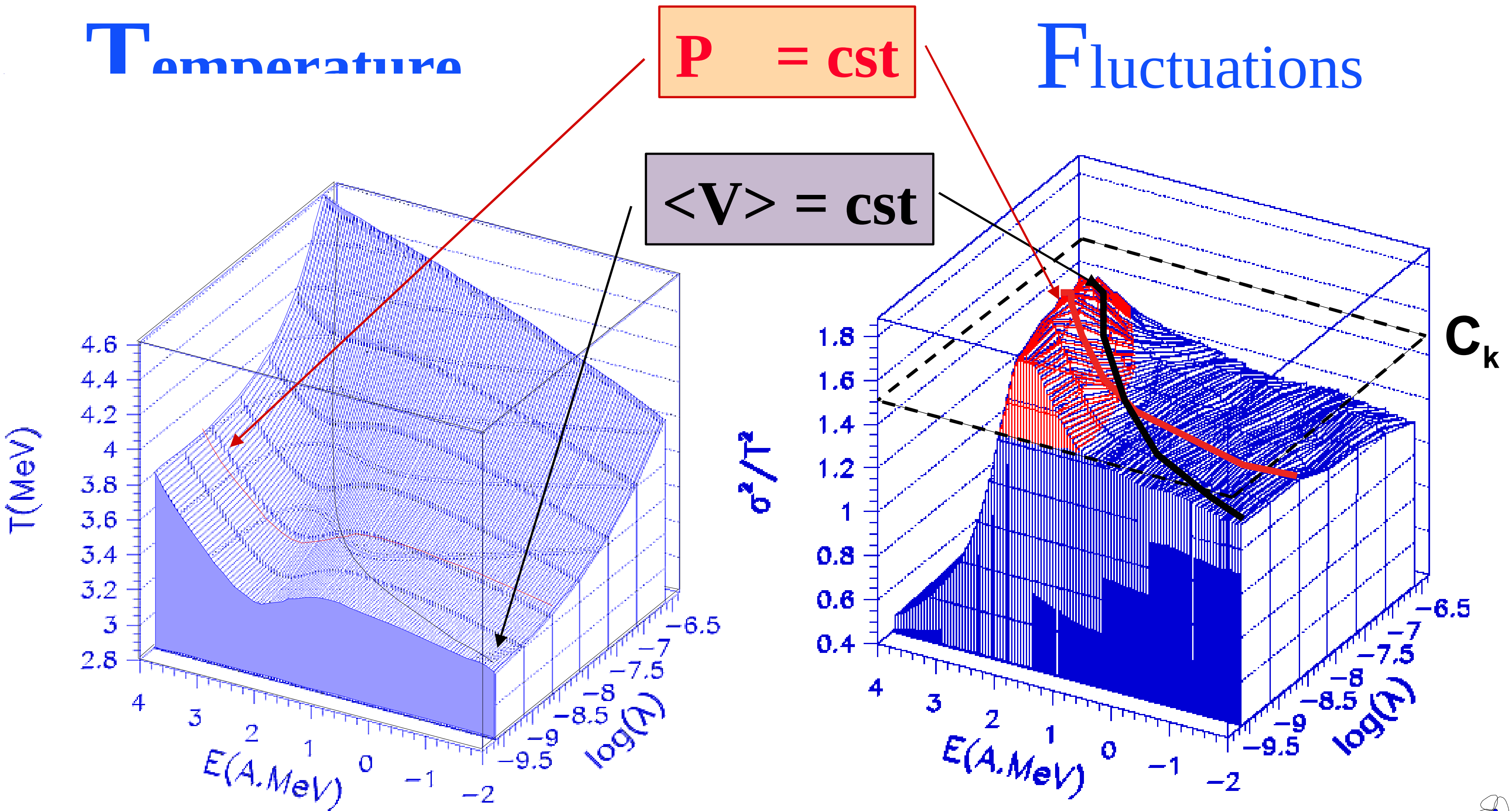
  - P = cst

  - <V> = cst

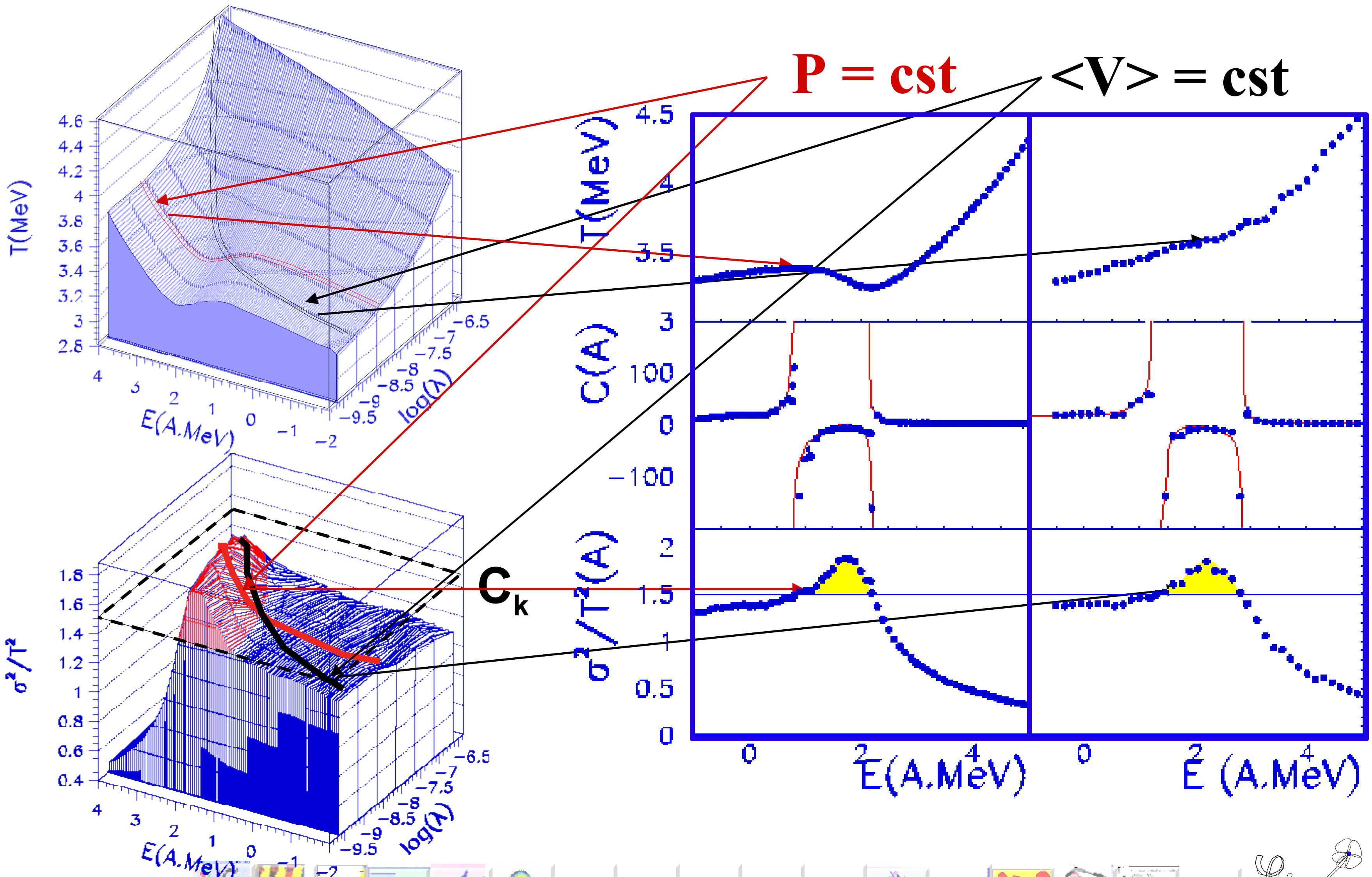
- Negative Heat Capacity



Volume dependent EOS

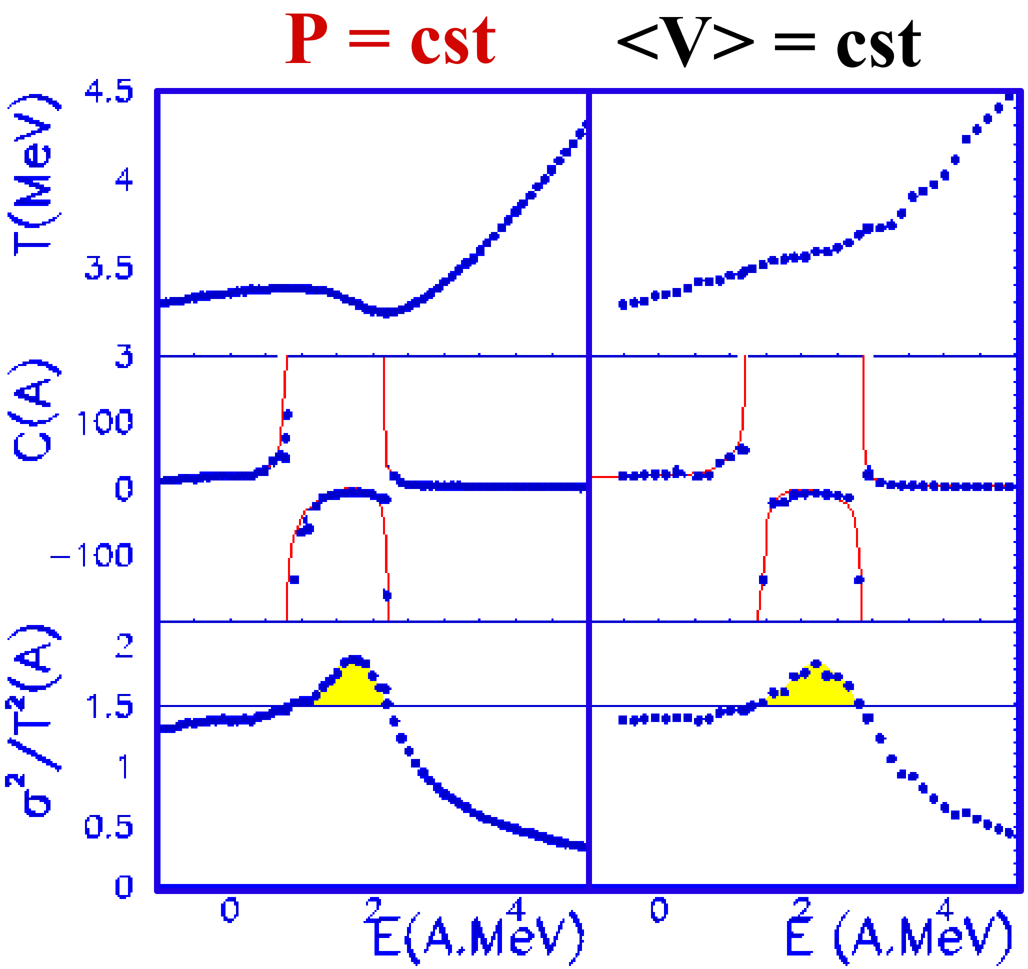


Volume dependent EOS



Volume dependent EOS

- The Caloric curve depends on the transformation
- **C is not dT/dE**
- **σ^2 measures EOS Abnormal in Liquid-Gas**



Caloric curve are not EOS
Fluctuations are

- The Caloric curve depends on the transformation

- **C is not dT/dE**

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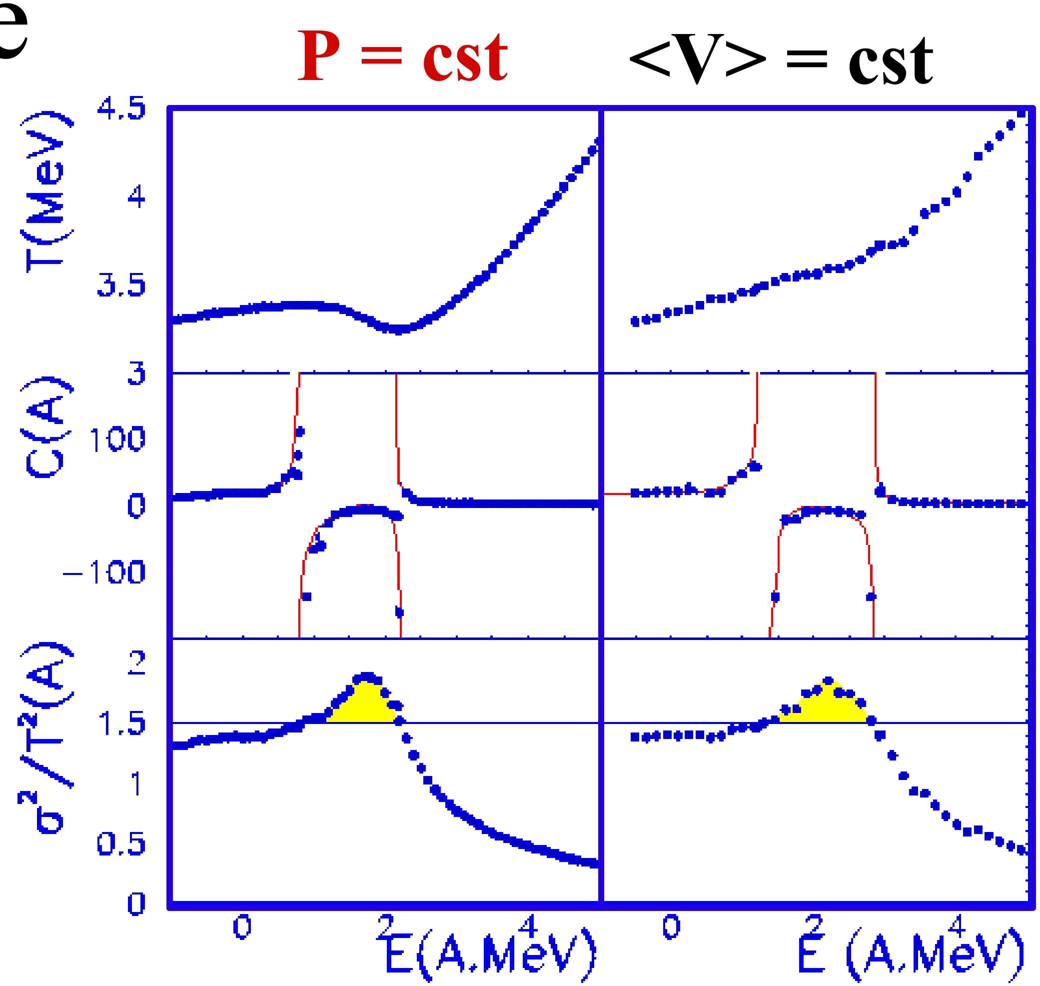
  σ^2 abnormal in Liquid-Gas



- c -

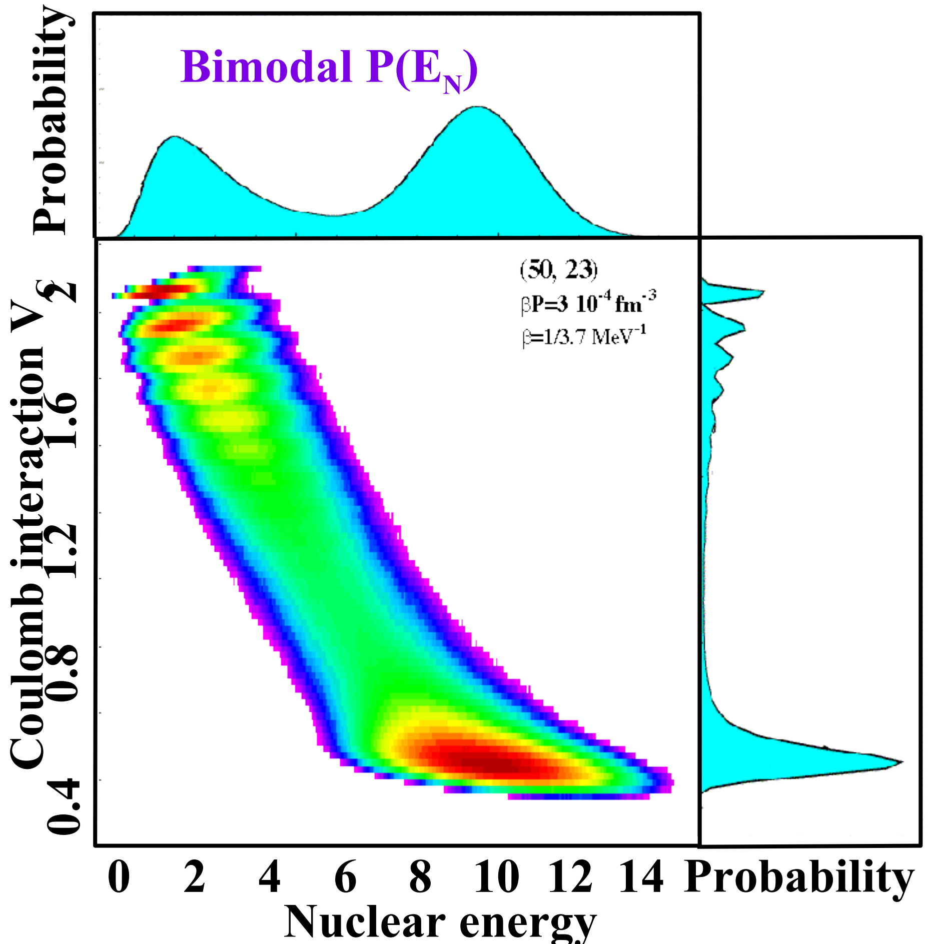
Negative curvatures

Channel opening

Ex: Statistical models

C<0 in statistical models
Liquid-gas and channel opening

- ◆ q = 0 => no-Coulomb
- Many channel opening
- One Liquid-Gas transition (Vc related to V)



-Appendix -V-

How to progress

- Correcting errors in σ reconstruction
  - Iterative procedure
- C from other observables
  - Using correlated observables

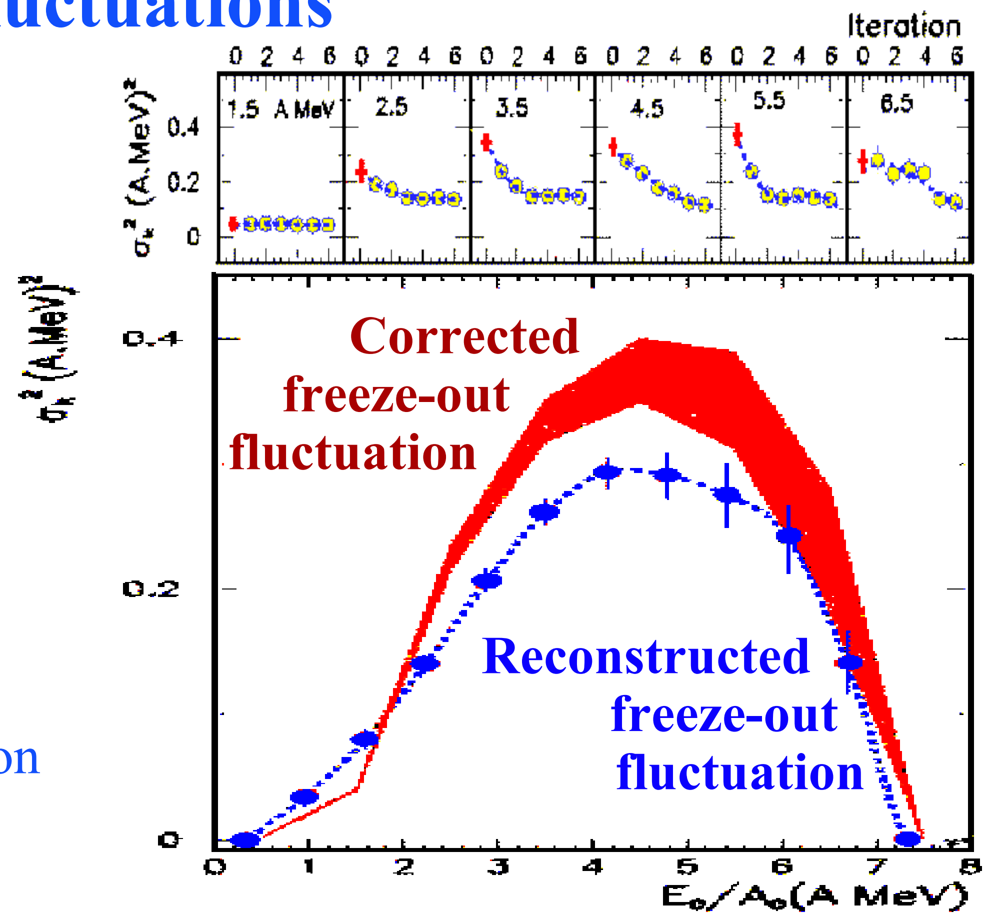
Correction of experimental errors

- **Heat capacity from fluctuations**
  -
- σ_{can} can be "filtered"
  - (apparatus and procedure)

Correction of experimental errors

- **Heat capacity from fluctuations**
  -

- σ_{can} can be "filtered"
  - (apparatus and procedure)
- σ_k can be corrected
  - Iterate the procedure
    - Freeze-out reconstruction
    - Decay toward detector



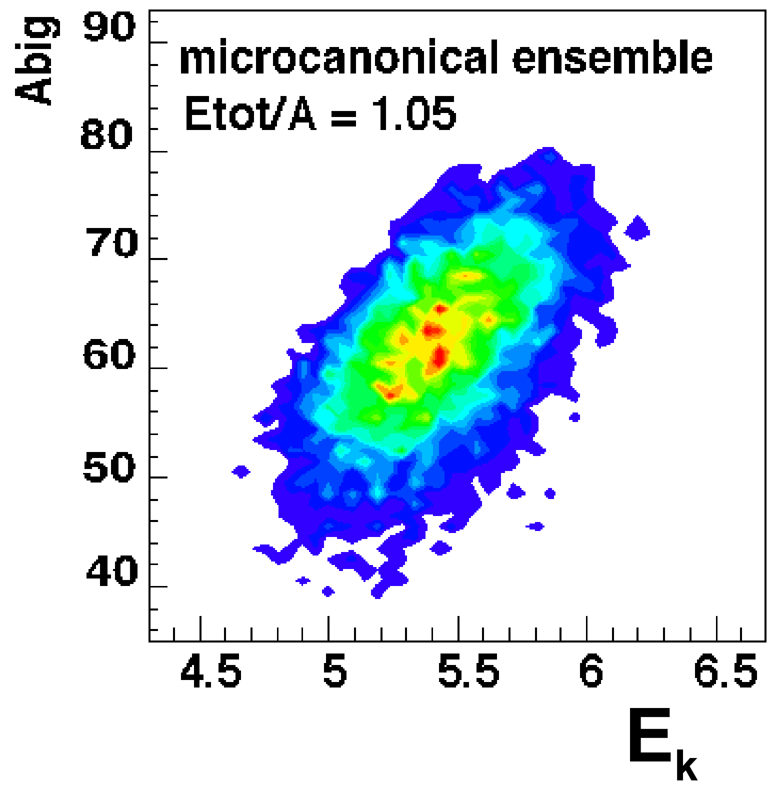
Heat capacity from any fluctuation

- From kinetic energy
  -

Heat capacity from any fluctuation

- **From kinetic energy**
  - ◆

- From any correlated observable (Ex. A_{big})
  - $$\frac{X_\kappa}{X} = 1 - \frac{\sigma_A^2}{\sigma_{A0}^2}\frac{1}{\rho^2}$$
  - $\rho = \sigma_{kA} / \sigma_k \sigma_A$ correlation
  - σ_{A0} "canonical" fluctuations

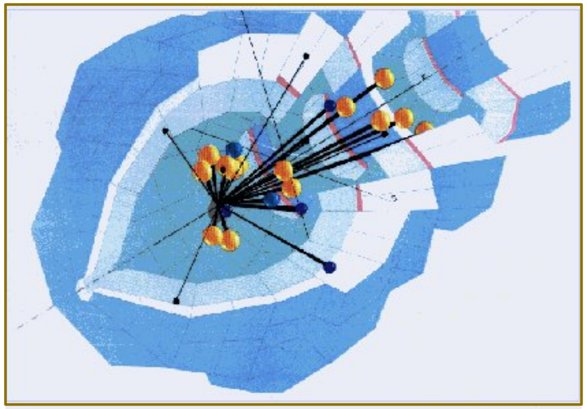


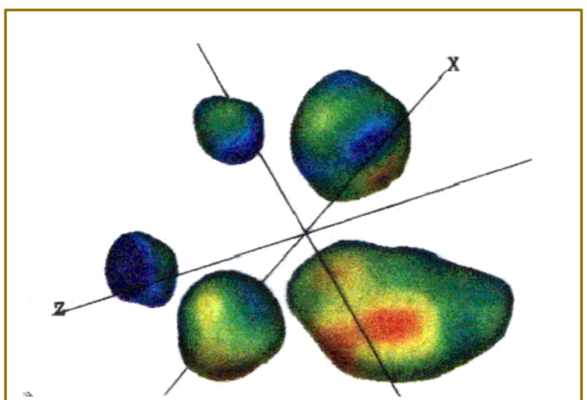
-Appendix -VI-

Test of freeze-out reconstruction

- Volume
  - Coulomb boost
- Temperature
  - Isotope ratios
  - Particle-fragment correlations
- Alternative extraction

Multifragmentation experiment





Sort events in energy (Calorimetry)

Reconstruct a freeze-out partition

1- Primary fragments: m_i

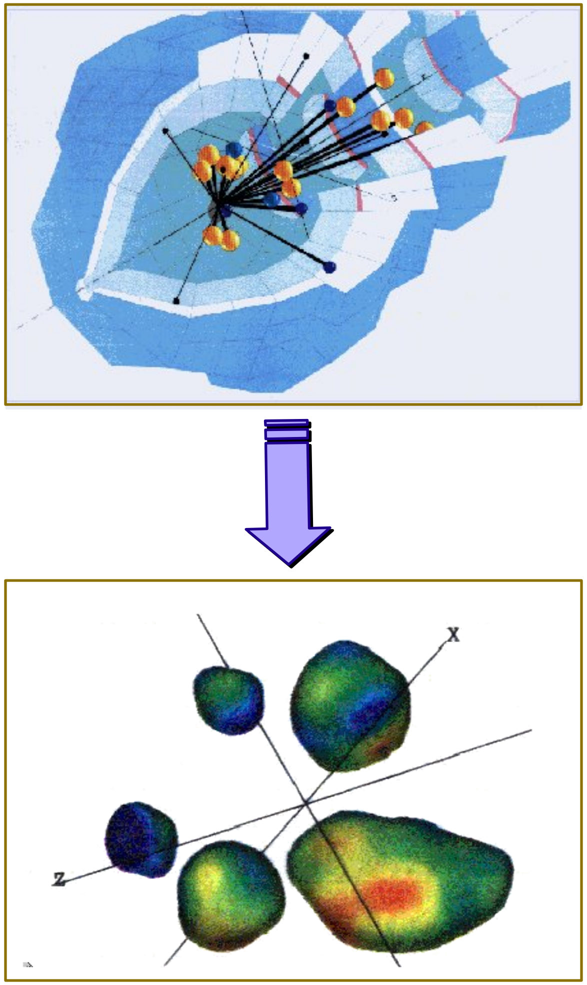
2- Freeze-out volume: E_{coul}

- $E_2 = \Sigma_i m_i + E_{coul}$
- $E_1 = E^* - E_2$ => $\langle E_1 \rangle, \sigma_1$

3- Kinetic EOS: T, C_1

- $\langle E_1 \rangle = \langle \Sigma_i a_i \rangle T^2 + 3/2 \langle M-1 \rangle T$

Multifragmentation experiment



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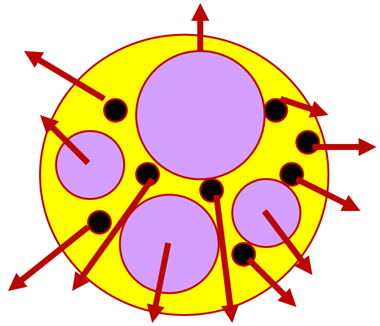
3- Kinetic EOS: T, C_1

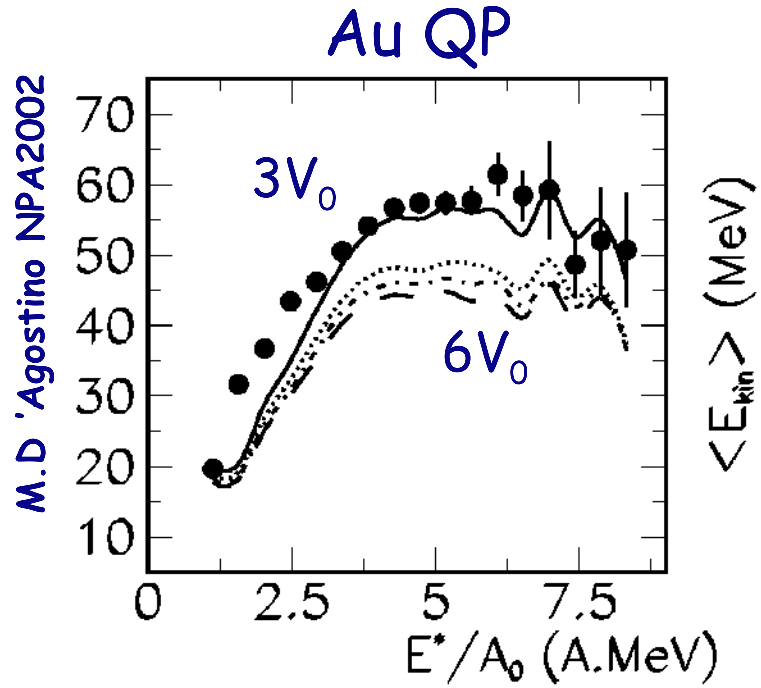
- $\langle E_1 \rangle = \langle \Sigma_i a_i \rangle T^2 + 3/2 \langle M-1 \rangle T$

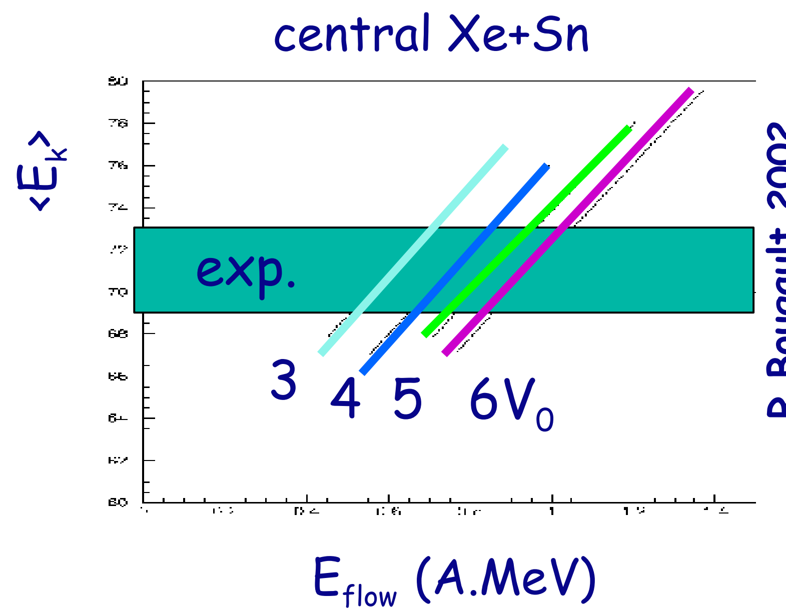
E*

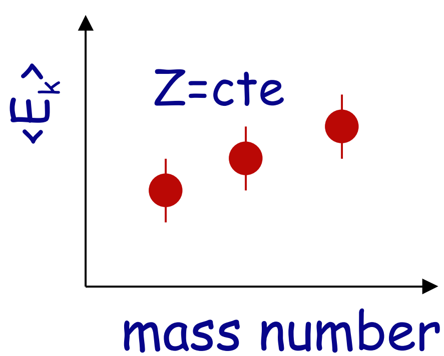
Volume



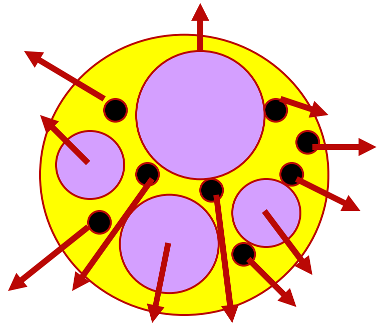
- From Coulomb boost
  - **C Weakly sensitive to V**
- Ambiguity with expansion
  - Need isotopic resolution
  - Tiny influence on C







Volume



- From Coulomb boost
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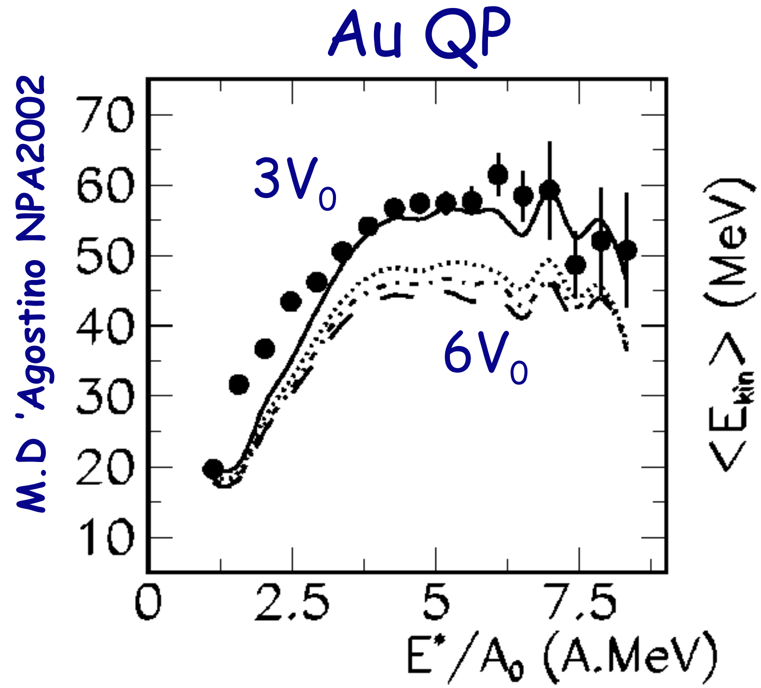
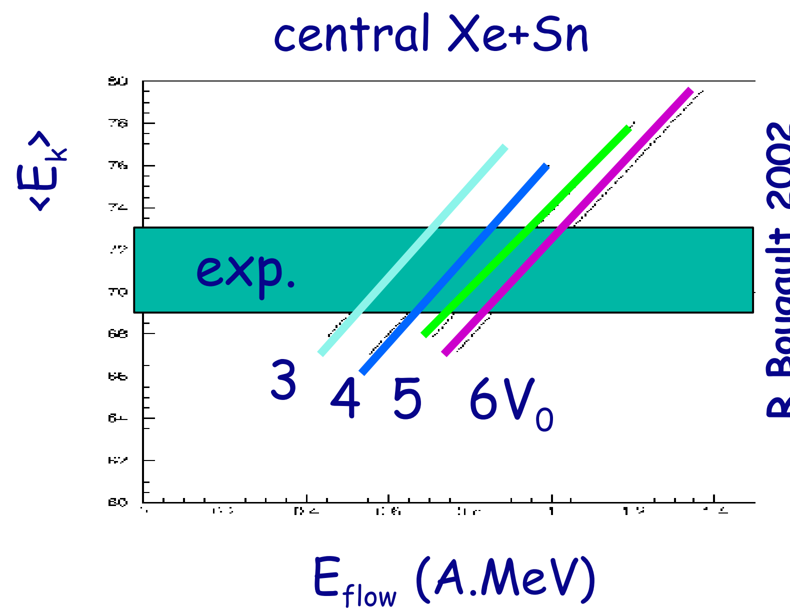
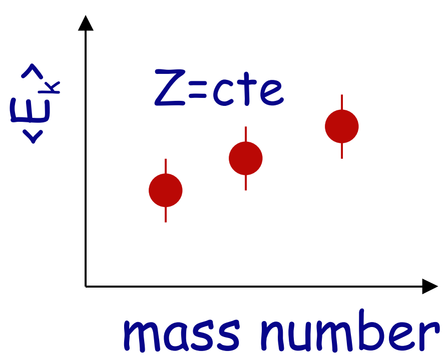






Influence on C

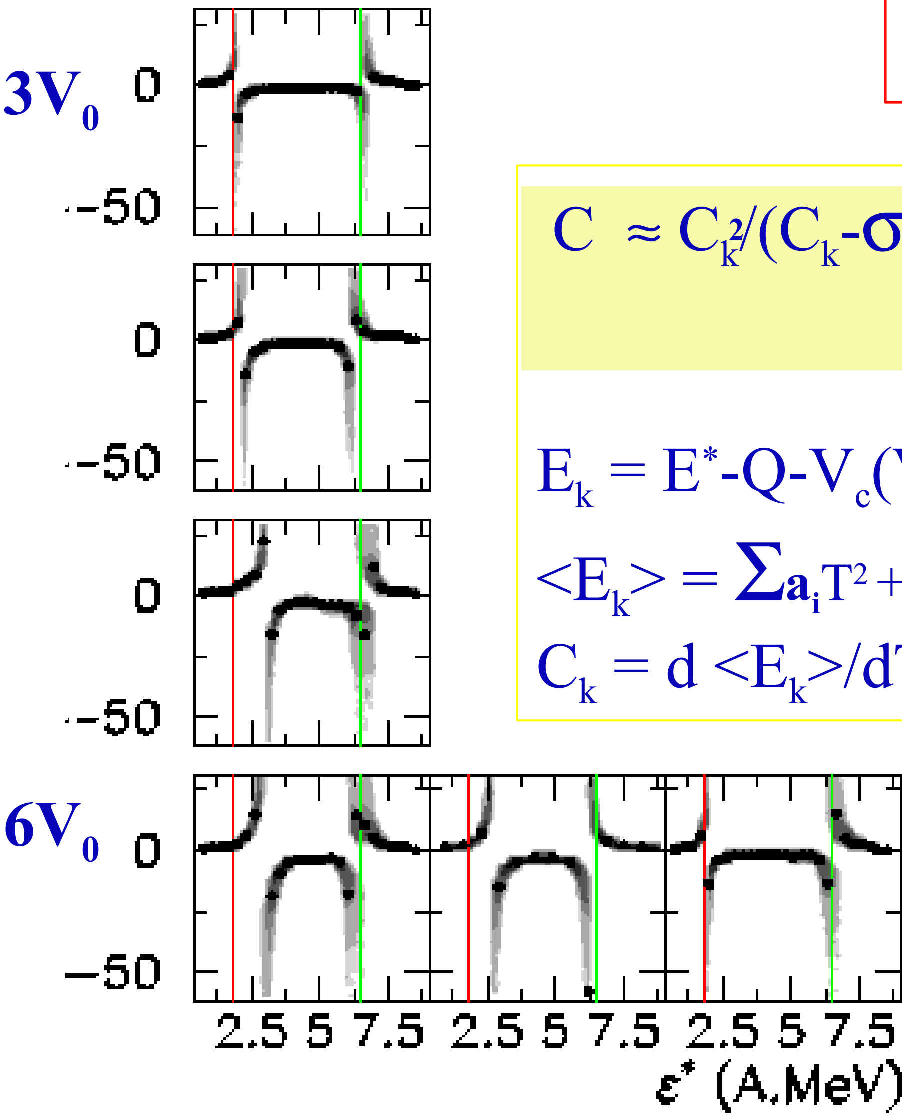
Au+Au 35 A.MeV
MULTICS data

$C \approx C_k^2/(C_k - \sigma_k^2/T^2)$

$E_k = E^* - Q - V_c(\mathbf{V})$

$\langle E_k \rangle = \sum \mathbf{a}_i T^2 + 3/2(\langle m \rangle - 1)T$

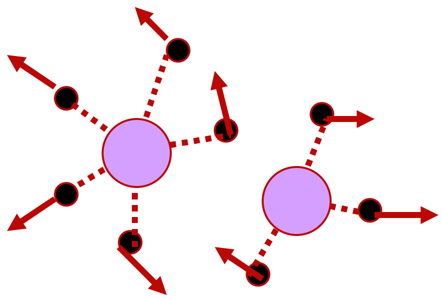
$C_k = d\langle E_k \rangle/dT \qquad C_k < 1.5\ A_{tot}$



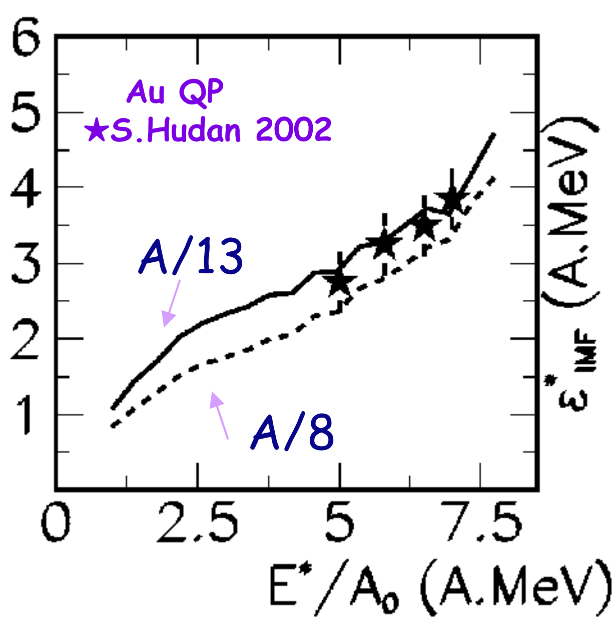
a=12 → a=8

M.D'Agostino et al. 2001

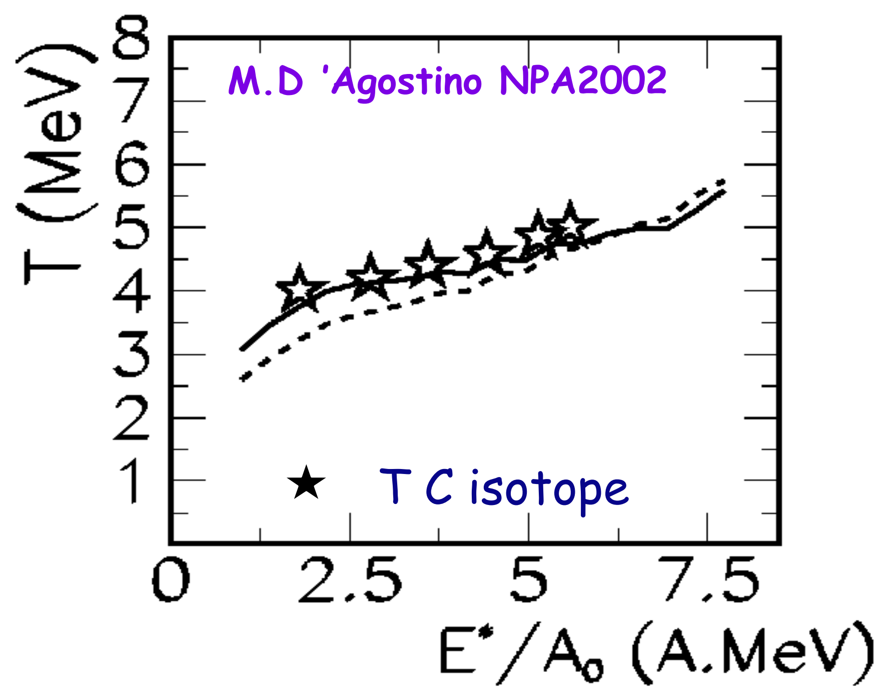
Evaporation and temperature



- **Correlation**



$$\langle E_k\rangle = \sum a_i(T)T^2 + 3/2(\langle m\rangle-1)T$$



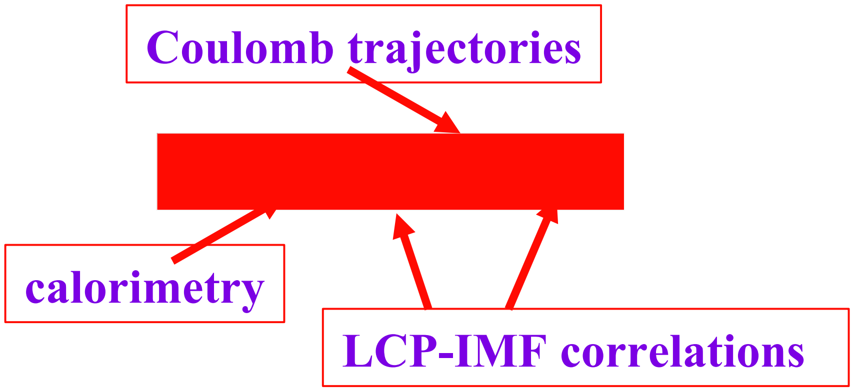
- Consistency of T

Heat capacity from the translational energy

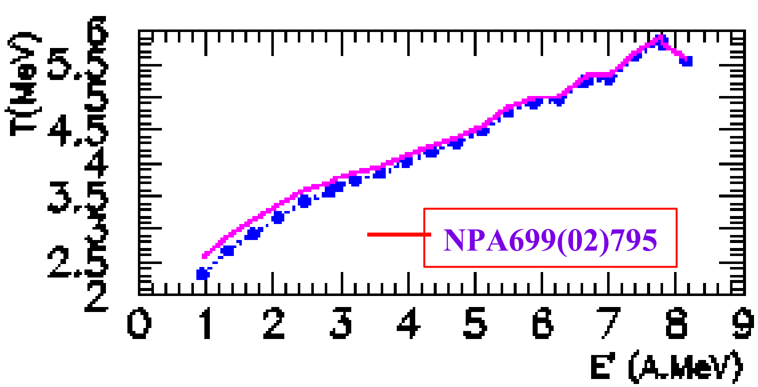
$$T = \left\langle \frac{E_{tr}}{(3/2)m - 1} \right\rangle$$

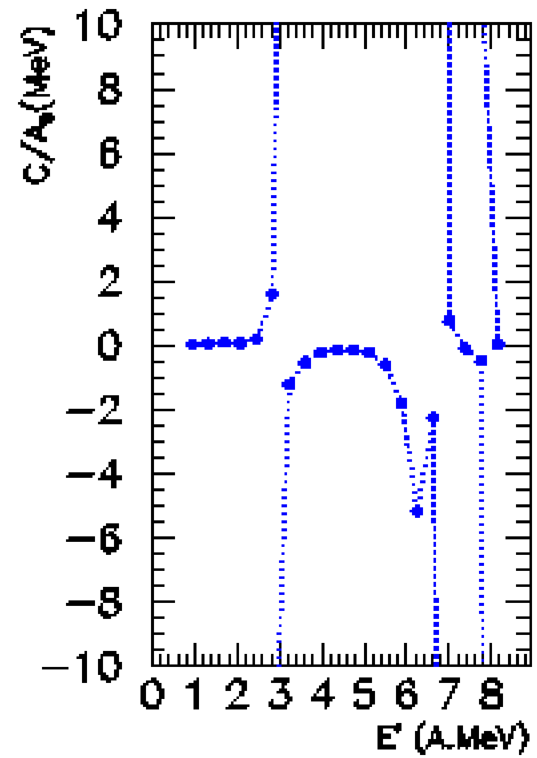
$$\frac{1}{C} = 1 - T^2 \left\langle \frac{\left((3/2)m - 1\right)\left((3/2)m - 2\right)}{E_{tr}^2} \right\rangle$$

E.M.Pearson 1985, A.Raduta 2002

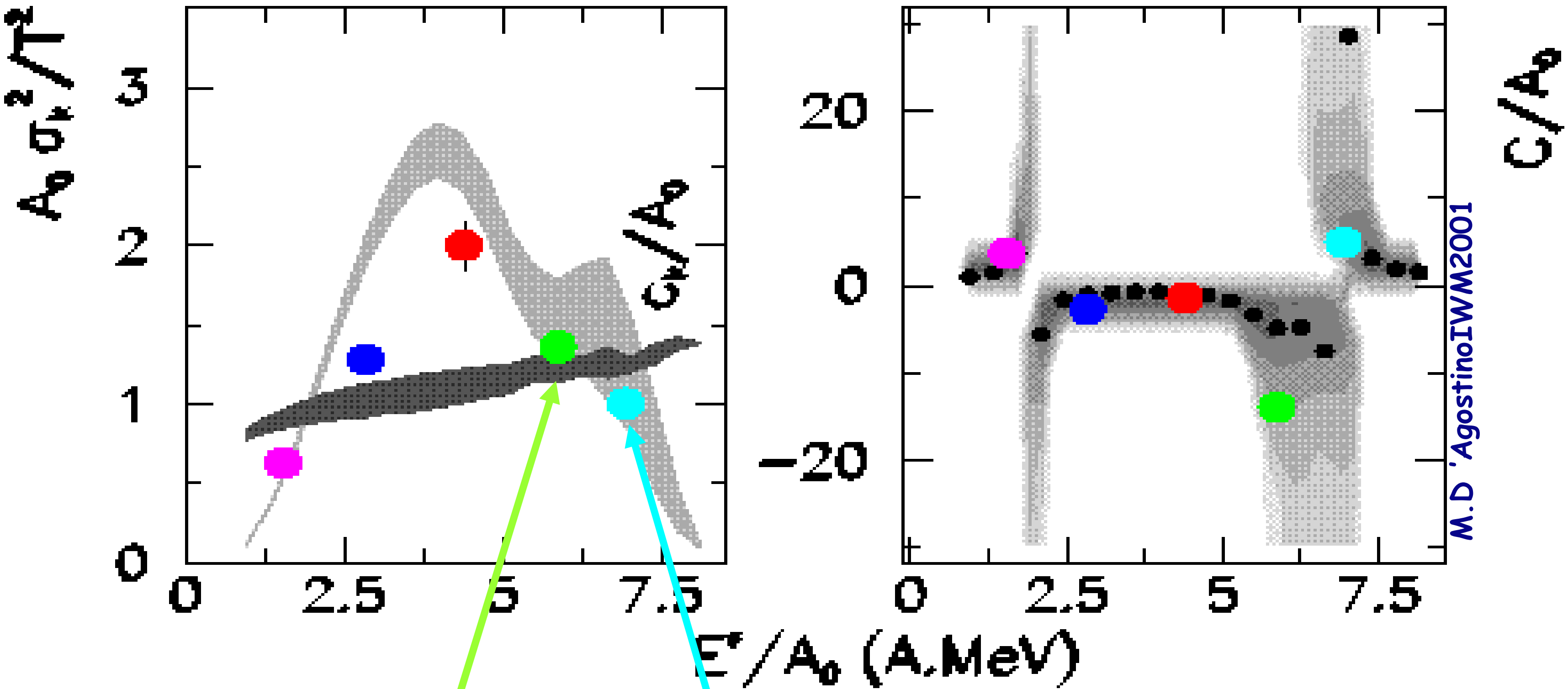


Multics
Au+Au 35A.MeV





Comparison central/peripheral



Au+Au 35 E_F=1

Au+Au 35 E_F=0

Up to ≈ 35 A.MeV the flow ambiguity is a small effect