Gauge-Higgs Unification

— An approach to beyond the standard model —

Kazunori TAKENAGA (Tohoku Univ.)

$28 \sim 30$ June, 2006 at the 4th COE Symposium, Sendai, JAPAN

Collaborators;
N. Haba (Tokushima(Munich)), Y. Hosotani (Osaka), N. Maru(Roma), M. Sakamoto (Kobe), T. Yamashita (SISSA)

I. Standard Model

Players in the standard model;

⋆ Matter

Quarks			Leptons		
u	c	t	ν_e	ν_μ	ν_τ
d	s	b	e	μ	τ

⋆ Gauge particles

g,	$W^\pm$,	Z,	γ

Quantum field theory with chiral gauge symmetry $\boxed{SU(3)_c \times SU(2)_L \times U(1)_Y}$

$SU(3)_c$ $\cdots$ strong interaction (nuclear force, QCD)

$SU(2)_L \times U(1)_Y$ $\cdots \begin{cases} \text{weak interaction } (\beta \text{ decay}) \\ \text{electromagnetic interaction (QED)} \end{cases}$

Quantum number

Players	spin	$SU(3)_c$, $SU(2)_L$, $U(1)_Y$
g W B	1	$(\mathbf{8}, \mathbf{1}, 0)$ $(\mathbf{1}, \mathbf{3}, 0)$ $(\mathbf{1}, \mathbf{1}, 0)$
$Q_L = (u, d)_L$ u_R d_R $L_L = (\nu_e, e)_L$ e_R (ν_R)	$\frac{1}{2}$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ $(\mathbf{3}, \mathbf{1}, \frac{2}{3})$ $(\mathbf{3}, \mathbf{1}, \frac{-1}{3})$ $(\mathbf{1}, \mathbf{2}, \frac{-1}{2})$ $(\mathbf{1}, \mathbf{1}, -1)$ $(\mathbf{1}, \mathbf{1}, 0)$
Φ (Higgs)	0	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

Higgs mechanism; $SU(3)_c \times SU(2)_L \times U(1)_Y \overset{\langle\Phi\rangle \neq 0}{\Longrightarrow} SU(3)_c \times U(1)_{em}$

mass for
- gauge particles $W^{\pm}$, $Z \leftarrow$ (one of the mixing between W and B)
- fermions, Higgs(not found yet) ($m_\Phi \gtrsim 114$ (GeV) by LEP I, II at CERN)

consistent with almost all the experiments

⋆ We expect the Higgs is found at LHC ('07) at CERN

Shortcoming

- many free parameters, 18
  3(gauge couplings) $+\, 9$(quark, lepton masses) $+\, 4$(KM matrix) $+\, 2$(Hiss sector)

- no predictability in the Higgs sector
  Higgs mass, self-interactions,$\cdots$

- fine tuning problem in the Higgs sector (hierarchy problem)
  quadratically divergent quantum corrections for the Higgs mass, $\delta m_H^2 \sim \Lambda^2$
  (suffered from UV effects, fine tuning of $(10^{-17})^2$ for $\Lambda = M_p \sim 10^{19}$ (GeV))

$\Longrightarrow$ beyond the standard model

Namely, ●● $\Longrightarrow$ lack of symmetry principle

- Supersymmetry (SUSY): (bosons $\leftrightarrow$ fermions)

  allowed symmetry in quantum field theory,
  the last symmetry (in a sense) $\because$ Coleman-Mandula, Haag-Lopuszanski-Sohnius)

  superparticles $\cdots$ $\tilde{e}$ (selectron) $\leftrightarrow e$, $\tilde{\gamma}$ (photino) $\leftrightarrow \gamma$

  $\Longrightarrow \delta m_H^2 = 0$, no quadratic divergence ($\stackrel{\text{SUSY breaking}}{\Rightarrow}$ $\delta m_H^2 \sim O(v^2)$)

  ⋆ light Higgs boson $\lesssim 130$ GeV

  ⋆ gauge coupling unification

  ⋆ dark matter candidate, *et.al.*

We would like to consider **Higher dimensional gauge symmetry**

$\Downarrow$

Gauge-Higgs unification

II. Gauge-Higgs unification; unification of spin 1 and spin 0 fields in $A_{\hat{\mu}}$

We consider a gauge theory in higher dimensional space-time

$M^D \to M^4 \times$ (compact manifold) (extra dimensions are compactified)

$$A_{\hat{\mu}} \overset{\text{compactification}}{\Longrightarrow} \begin{cases} A_\mu \to 4\text{Dim. gauge bosons} + \text{KK modes}, \\ A_i \to 4\text{Dim. scalars (the Higgs)} + \text{KK modes} \end{cases}$$

Why is this attractive ?

⋆ The D-dim. gauge symmetry forbids the divergence for the Higgs mass
⇒ finite Higgs mass (no UV effect on the Higgs mass)
⇒ resolution to the hierarchy problem

⋆ The D-dim. gauge interaction governs the Higgs self-interactions
⇒ no free parameter in the Higgs sector (↔ model building)
⇒ predictability in the Higgs sector

¶ What does the gauge-Higgs unification look like ?

A simple example; 5 dimensional QED on $M^4 \times S^1$

$$S = \int_0^{2\pi R} dy \int d^4x \;\; \frac{-1}{4} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} + \bar{\Psi} i \Gamma^{\hat{\mu}} \left(\partial_{\hat{\mu}} - i g A_{\hat{\mu}}\right) \Psi$$

Specify boundary conditions (B.C.'s) of fields for compactified directions

$$\left(\mathcal{L}(x, y+L) = \mathcal{L}(x, y); \quad \text{singlevalueness}, \quad L = 2\pi R \right)$$

We choose Periodic B.C. $A_{\hat{\mu}}(x, y+2\pi R) = A_{\hat{\mu}}(x, y), \quad \Psi(x, y+2\pi R) = \Psi(x, y).$

Mode expansion $\cdots A_{\hat{\mu}}(x, y) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} A_{\hat{\mu}}^{(n)}(x) \, \mathrm{e}^{i\frac{2\pi n}{L}y}, \quad n$: Kaluza-Klein mode

$$
\begin{aligned}
S_{gauge} \;&=\; \int d^4x \;\; \frac{-1}{4} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \frac{1}{2}\partial_\mu A^{(0)}_y \partial^\mu A^{(0)}_y \\
&+\; \sum_{n\neq 0} \frac{-1}{4} F^{(n)*}_{\mu\nu} F^{\mu\nu(n)} + \frac{1}{2}\left(\frac{2\pi n}{L}\right)^2 \left| A^{(n)}_\mu + \frac{iL}{2\pi n}\partial_\mu A^{(n)}_y \right|^2
\end{aligned}
$$

$A^{(0)}_\mu \cdots$ a massless gauge field, $A^{(0)}_y \cdots$ a "massless" real scalar field,

$A^{(n)}_\mu \cdots$ massive spin one field, $A^{(n)}_y \cdots$ unphysical mode (eaten by $A^{(n)}_\mu$)

The zero mode $A^{(0)}_y$: physical scalar field

Importance of the gauge invariance;

5 dim. gauge transformation in terms of 4 dim. language

$$A_\mu^{(0)}(x) \to \quad A_\mu^{(0)}(x) + \partial_\mu \theta^{(0)}(x),$$
$$A_y^{(0)}(x) \to \quad A_y^{(0)}(x) + \frac{2\pi}{\bar{g}L} m, \qquad (\bar{g} \equiv \frac{g}{\sqrt{L}}) \quad \cdots (1) \ \text{shift symmetry}$$

(1) implies that
- (A) $\bar{g}L\langle A_y^{(0)}\rangle = 2\pi m \quad \leftrightarrow \quad \langle A_y^{(0)}\rangle = 0$; gauge equivalent
- (B) $\bar{g}L\langle A_y^{(0)}\rangle = \theta(\neq 2\pi m) \quad \leftrightarrow \quad \langle A_y^{(0)}\rangle = 0$; inequivalent

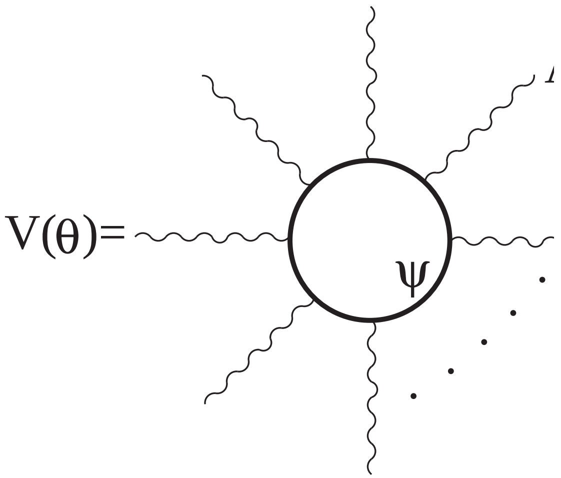
↖ ↗
physically distinct configuration (θ; mod. 2π)

The constant θ cannot be gauged away, and the VEV is physically meaningful, reflecting the topology of S^1.

generalized to nonabelian gauge theory

Aharanov-Bohm effect; $\Phi = B \times \pi R^2 \Longleftrightarrow g \oint_{S^1} dy \langle A_y \rangle = \theta$

- similarity; interference pattern by $\Phi \Longleftrightarrow \theta$; spectrum of the theory
- difference; a parameter, $\Phi \Longleftrightarrow \theta$ determined by the effective potential for θ



$M^4 \times S^1$

$G = U(1)$, fermion contributions (P.B.C.)

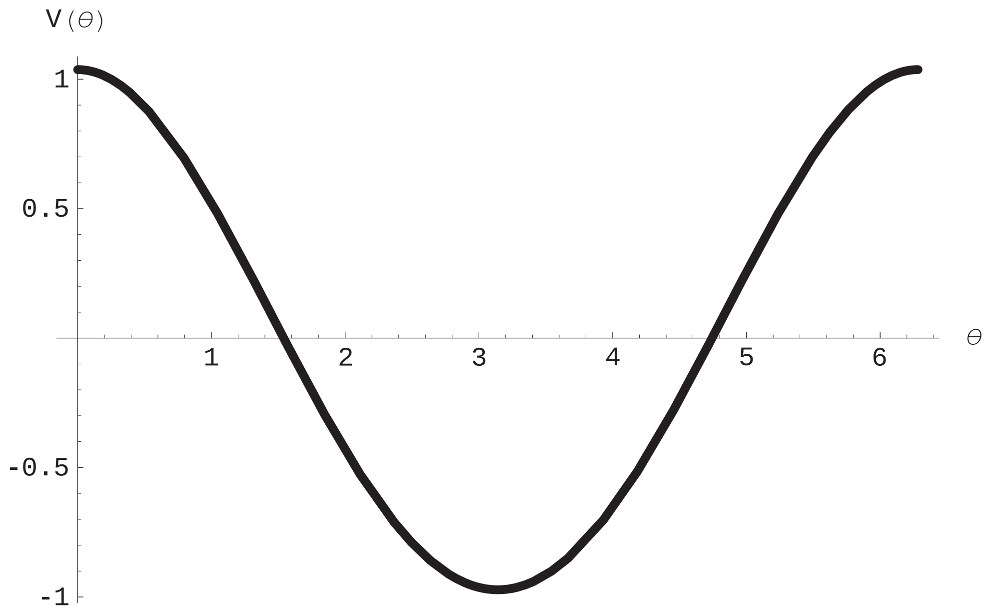
$$V(\theta) = \frac{\Gamma(5/2)}{\pi^{5/2}} \times (4N_F) \times \frac{1}{L^5} \times \sum_{m=1}^{\infty} \frac{1}{m^5} \cos(m\,\theta) + \cdots$$

- Summing up all the K-K modes, n (shift symmetry ; $A_y \to A_y + 2\pi m/gL$)

original gauge invariance, quantum corrections in extra dimension

The behavior of

$$\sum_{n=1}^{\infty} \frac{1}{n^5} \cos(n\theta) = \mathsf{Re}\ \mathsf{Li}_5(\mathrm{e}^{i\theta}), \quad \text{where} \quad \mathsf{Li}_D(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^D} \quad \text{(Polylogarithmic function)}$$



- $V_{eff}(\theta)$; radiatively generated,

only gauge invariant; $W = \exp(ig \oint_{S^1} dy\ A_y)$ the Wilson line phase

$\Rightarrow V_{eff}(\theta) = F_{eff}(W)$; nonlocal form $\Rightarrow$ no UV effect on the potential $\Rightarrow$ finite

$\frac{\partial V_{eff}(\theta)}{\partial \theta} = 0 \Rightarrow \theta = \theta_0 = \pi$; dynamically determined (no free parameter in the scalar sector)

- $\left.\frac{\partial^2 V_{eff}(\theta)}{\partial \theta^2}\right|_{\theta=\theta_0} \sim$ mass term for $A_y^{(0)}$

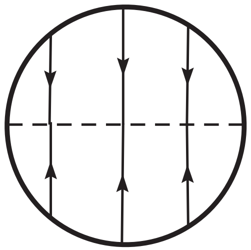
$$m_{A_y}^2 = (gL)^2 \left.\frac{\partial^2 V_{eff}(\theta)}{\partial \theta^2}\right|_{\theta=\theta_0=\pi} = 3N_F \zeta(3) \times \frac{\bar{g}^2}{L^2} \qquad \text{(finite)}$$

$A_y^{(0)}$ (scalar) becomes massive after taking into account quantum corrections.

III. Toward a realistic model

5 dim. model $M^4 \times S^1/Z_2$ (an orbifold compactification)

identificatuions; $S^1 \cdots y \sim y + 2\pi R,\ Z_2 \cdots y \sim -y$



$\Rightarrow 0 \leq y \leq \pi R$ with two fixed points

start with $SU(3)$
gauge group $A_{\hat{\mu}} = A^a \frac{\lambda^a}{2} = (A_\mu,\ A_y)$

$$A_\mu = \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ \hline (-,-) & (-,-) & (+,+) \end{array}\right)$$

zero modes $(+,+)$ $\Rightarrow SU(2) \times U(1)$,

$$A_y = \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ \hline (+,+) & (+,+) & (-,-) \end{array}\right)$$

zero mode $(+,+)$ $\Rightarrow$ an $SU(2)$ doublet $=$ Higgs

$$\Phi \equiv \sqrt{2\pi R}\,\frac{1}{\sqrt{2}}\begin{pmatrix} A_y^4 - iA_y^5 \\ A_y^6 - iA_y^7 \end{pmatrix} \Rightarrow \langle\Phi\rangle = \sqrt{2\pi R}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \langle A_y^6\rangle \end{pmatrix},$$

where $\langle A_y^6 \rangle = \dfrac{a}{gR} = v$

The kinetic term for the Higgs,

$$-\frac{1}{2}\text{tr}\left(F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}\right) \stackrel{(\hat{\mu},\hat{\nu})=(\mu,y)}{\Longrightarrow}$$

$$\int_0^{2\pi R} dy \ -\text{tr}\left(F_{\mu y}F^{\mu y}\right) = \left|\left(\partial_\mu - i\bar{g}A_\mu^{a(0)}\frac{\tau^a}{2} - i\frac{(\sqrt{3}\bar{g})}{2}A_\mu^{8(0)}\right)\Phi\right|^2; \quad SU(2)\times U(1)$$

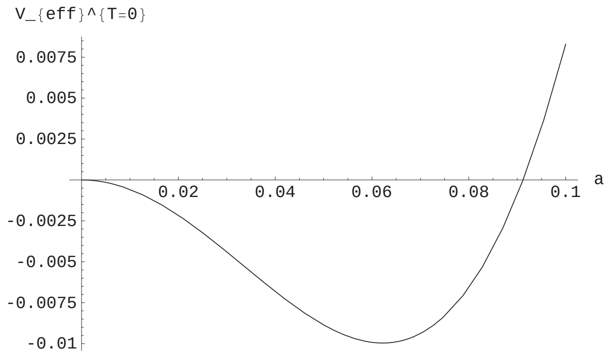
$\star$ $\mathbf{8} \in SU(3) = \mathbf{3}_0 \oplus \mathbf{2}_{1/2} \oplus \mathbf{2}^*_{-1/2} \oplus \mathbf{1}_0, \quad Q_Y \equiv \frac{1}{\sqrt{3}}\frac{\lambda^8}{2} = \text{diag.}(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3})$

It is possible to identify the Higgs field with a part of the component gauge field in higher dimensional gauge field.

What has been known;

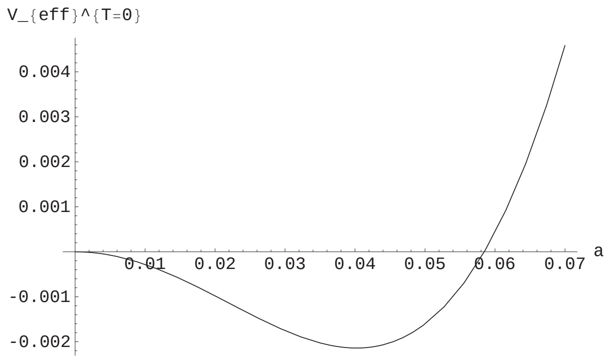
- The correct gauge symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{em}$ can be realized

[A]



$$(N_{adj}, N_{fd}, N^s_{adj}, N^s_{fd})^{(+)} = (2,3,0,1),$$
$$(N_{adj}, N_{fd}, N^s_{adj}, N^s_{fd})^{(-)} = (2,1,0,0),$$
$$a_0 \simeq 0.06, \quad U(1)_{em}$$

[B]



$$(N_{adj}, N_{fd}, N^s_{adj}, N^s_{fd})^{(+)} = (2,3,0,2),$$
$$(N_{adj}, N_{fd}, N^s_{adj}, N^s_{fd})^{(-)} = (2,1,0,1),$$
$$a_0 \simeq 0.04, \quad U(1)_{em}$$

- The size of the Higgs mass depends on the bulk matter. It is possible to obtain, for example, that

$$m_H \simeq 115, \quad 130 \text{ GeV}, \qquad (a \simeq 0.06, \quad 0.04)$$

which corresponds to $R^{-1} \simeq 4, \quad 6$ TeV $\Longrightarrow$ the size of the KK excitations new phenomena at around TeV region

What we have to do;

- constructing realistic models (under investigation)
  ⋆ the Weinberg angle, $\sin^2\theta_W = 3/4 \ \left(= \frac{(\sqrt{3}\bar{g})^2}{\bar{g}^2+(\sqrt{3}\bar{g})^2} \right)$

  ⋆ fermion mass spectrum ($\longleftarrow$ gauge interaction alone)

- studying new phenomena (KK excitations *et.al.*) expected in the scenario of the gauge-Higgs unification ($\Leftrightarrow$ experiments, LHC, ILC)