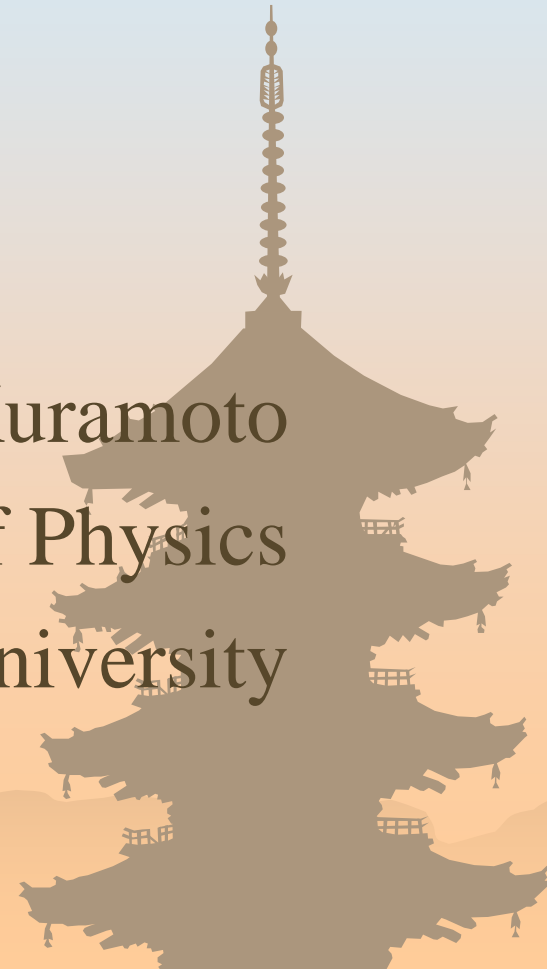


# Hierarchical structure in electronic multipole orders and fluctuations

Y. Kuramoto  
Department of Physics  
Tohoku University





# Our COE team for multipoles

- Sample preparation and thermodynamic measurements
  - H. Onodera, A. Tobo
- Neutron and X-ray scatterings
  - K.Iwasa, K.Ohoyama, L. Hao
- Theory
  - Y.Kuramoto, H.Kusunose, A.Kiss,
  - K. Kubo (JAERI), G.Sakurai (D3), H. Kono (D2), J.Otsuki (M2)

# Collective and dual behaviors of electrons



- Single electron
  - Known very well ?
- Interacting electrons
  - Unexpected consequences
  - Superconductivity, ferromagnetism, ...

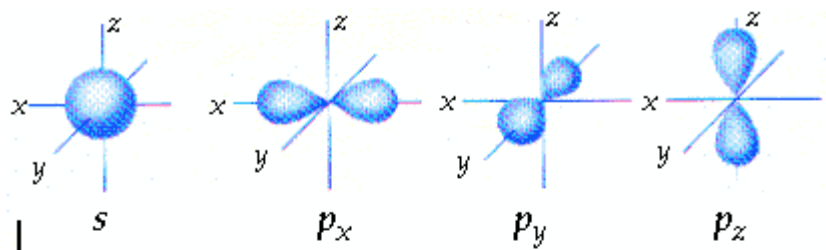


## Dual natures

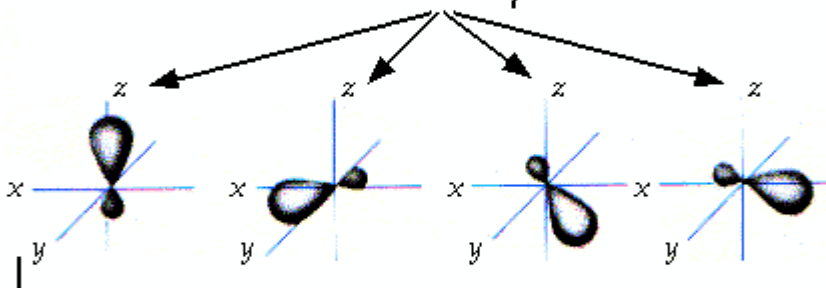
- Itinerant state :  $k$  ,  $\sigma$  (spin),  $v$  (band)
- Localized state + spin-orbit interaction
  - Orbital degeneracy => multipole moments



# Electron orbitals (in chemistry)



Combine to make four  $sp^3$  orbitals ...

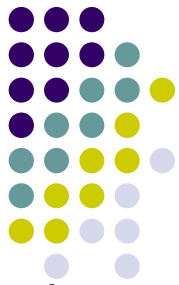


... which are represented by the set



$\text{CH}_4$  molecule

# Multipole moments

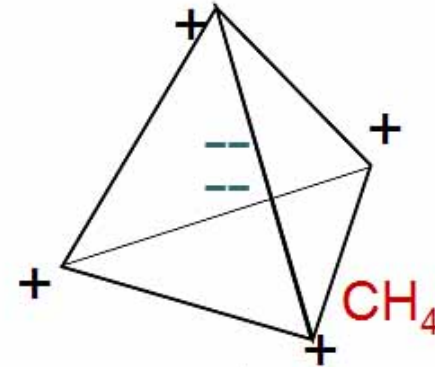
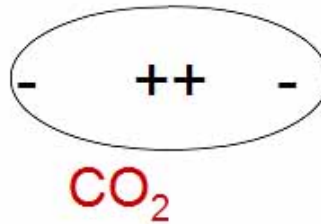
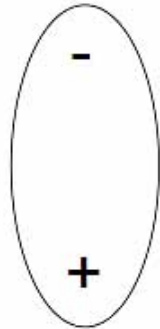


dipole

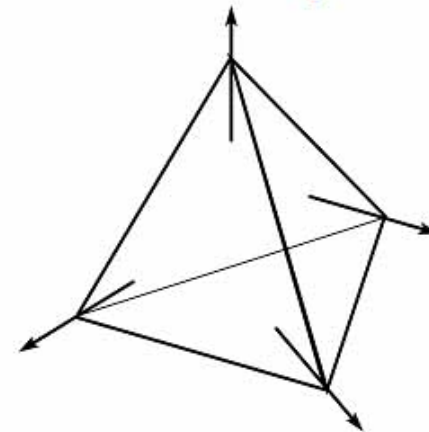
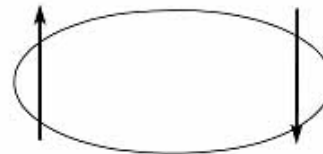
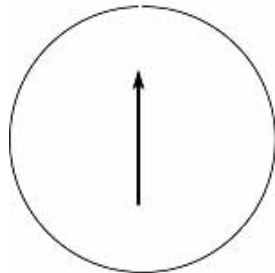
quadrupole

octupole (xyz)

electric

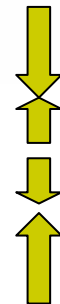


magnetic

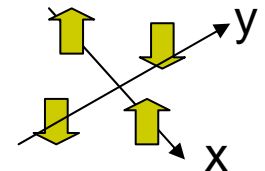


Other octupoles:

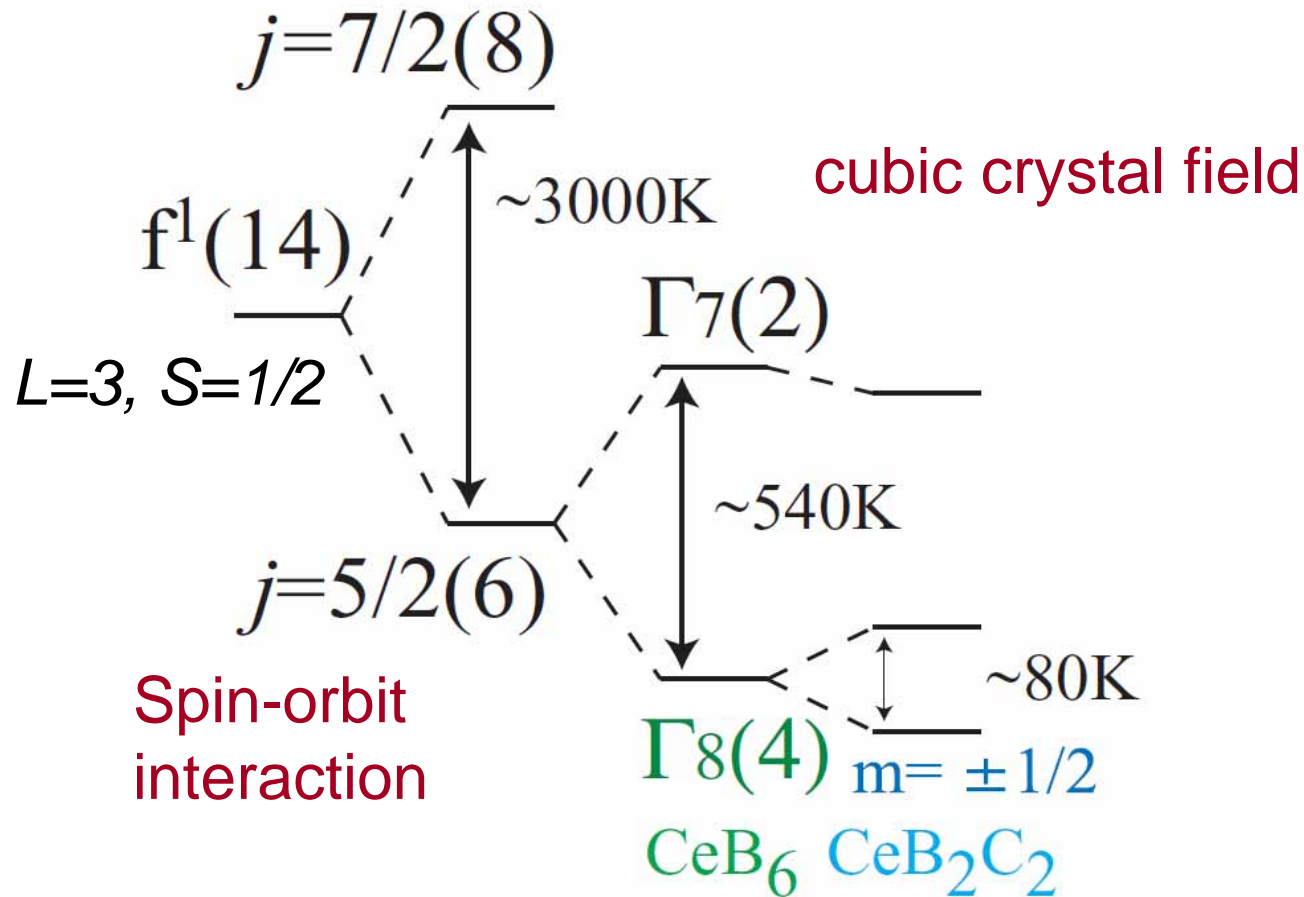
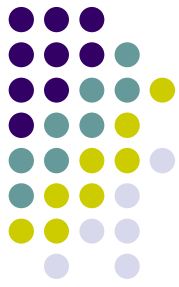
$$z(5z^2 - r^2)$$



$$z(x^2 - y^2)$$



# Splitting of $f^1$ energy levels with strong spin-orbit interaction

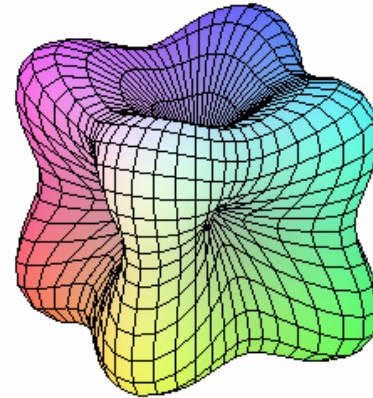


# Examples of $4f^1$ electron orbitals in cubic crystalline electric field (CEF) potential




$$L = 3, S = 1/2, J = 5/2$$

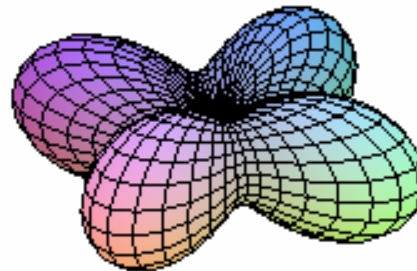
$\Gamma_7$    
Kramers doublet



$$|\uparrow\rangle = \sqrt{\frac{1}{6}}\left|+\frac{5}{2}\right\rangle - \sqrt{\frac{5}{6}}\left|-\frac{3}{2}\right\rangle$$

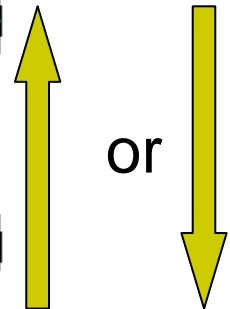
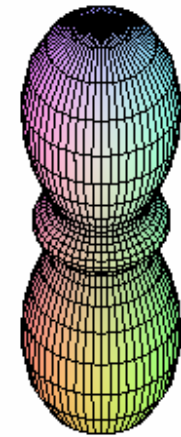
$$|\downarrow\rangle = \sqrt{\frac{1}{6}}\left|-\frac{5}{2}\right\rangle - \sqrt{\frac{5}{6}}\left|+\frac{3}{2}\right\rangle$$

$\Gamma_8$    
Kramers doublet  
- orbital doublet



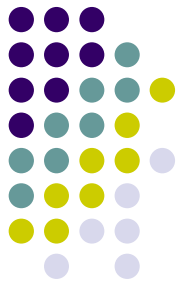
$$|+\uparrow\rangle = \sqrt{\frac{5}{6}}\left|+\frac{5}{2}\right\rangle + \sqrt{\frac{1}{6}}\left|-\frac{3}{2}\right\rangle$$

$$|+\downarrow\rangle = \sqrt{\frac{5}{6}}\left|-\frac{5}{2}\right\rangle + \sqrt{\frac{1}{6}}\left|+\frac{3}{2}\right\rangle$$



$$|-, \sigma\rangle = |\pm 1/2\rangle$$

# Hierarchical structure of internal magnetic field



- Dipole moment

$$H_{dip} \gg \mu_B / r^3 \gg 0.1 T$$

- Octupole moment

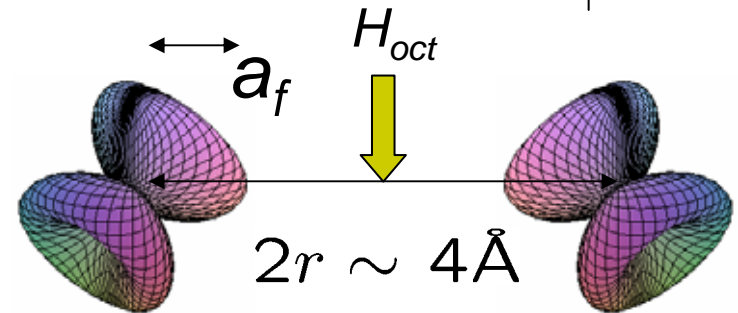
$$H_{oct} \gg \mu_B (a_f / r)^2 / r^3 \gg 10^{-1} H_{dip}$$

- **No hierarchy in intersite interactions**

- cf. Heisenberg theory of exchange interaction

$$J_{exch} \gg C e^2 / r \hat{=} \mu_B^2 / r^3 \gg \alpha^2 e^2 / r \gg 10^{-4} e^2 / r$$

- RKKY interaction => octupole order may be realized without dipole order!







# Role of higher multipoles

- Controlling the phase transition as **hidden order parameters**.
- Supporting unusual physical properties from behind.
- **Strong spin-orbit interaction** =>
  - Detectable by neutrons and X-rays?
  - Excitation spectrum of multipoles?
  - Interaction with conduction electrons?



# Modes of Research

- Neutron scattering (+polarization)
  - Tohoku University facilities at JAERI Tokai
- X-ray scattering by Synchrotron radiation
  - Spring-8 and Tsukuba
- Theory
  - Faithful account of electronic structure
  - Simplified models with essential physics

# 東北大学が所有または管理している装置 (JAERI at Tokai)



粉末回折装置**HERMES** (金研)  
結晶構造、磁気構造解析  
(希土類、3d酸化物、強相関係)



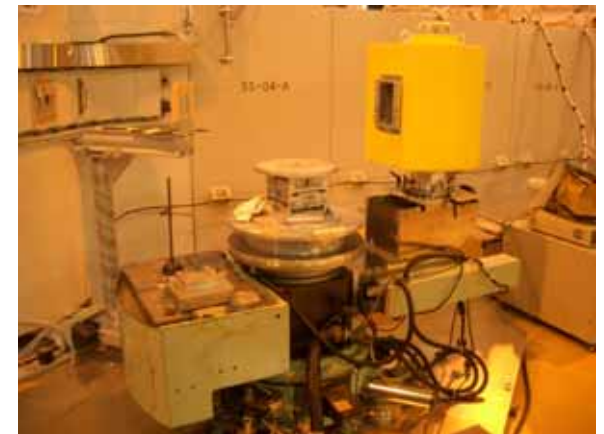
三軸分光器**TOPAN** (理学研究科)  
原子や磁気モーメントの運動・ゆらぎの観測  
(高温超伝導、誘電体、強相関係)



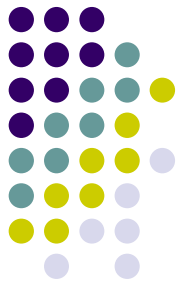
日本原子力研究所  
(茨城県東海村)



単結晶四軸回折装置**FONDER**  
(物性研所有: 多元研グループが管理)  
有機物など複雑な構造での構造解析



単結晶回折装置**KSD** (金研)  
(三軸分光器に改造中)



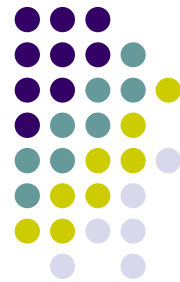
# Target systems

- Rare-earth hexaborides:
  - $\text{CeB}_6$ ,  $\text{Ce}_x\text{La}_{1-x}\text{B}_6$
- Rare-earth diborocarbides:
  - $\text{RB}_2\text{C}_2$  (R= Ce, Dy, Gd, Ho, Tb, Er...)
- Pr skutterudites:
  - $\text{PrFe}_4\text{P}_{12}$ ,  $\text{PrRu}_4\text{P}_{12}$ ,  $\text{PrOs}_4\text{Sb}_{12}\dots$

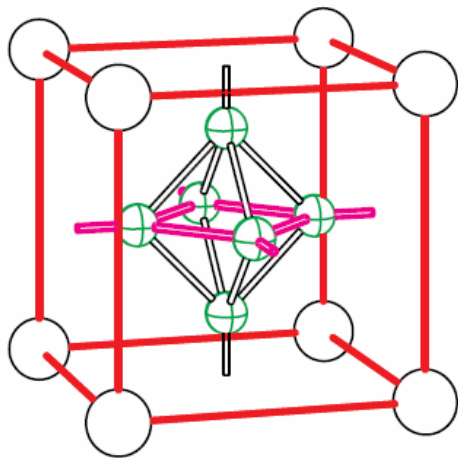
## Common structural features:

Clathrate-like lattices => cage+guest ions

# Hexaborides vs diborocarbides



$MB_6$  CsCl-type

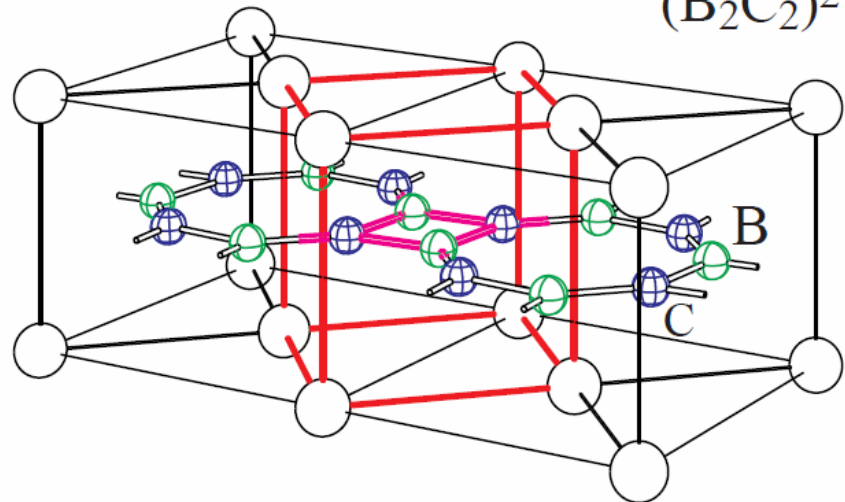


$B_6^{2-}$

M=La;  $a=4.156\text{\AA}$

M=Ca;  $a=4.1525\text{\AA}$

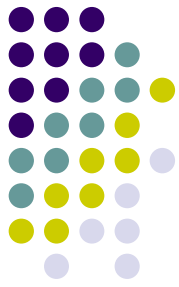
$MB_2C_2$  P4/mbm



$(B_2C_2)^{2-}$

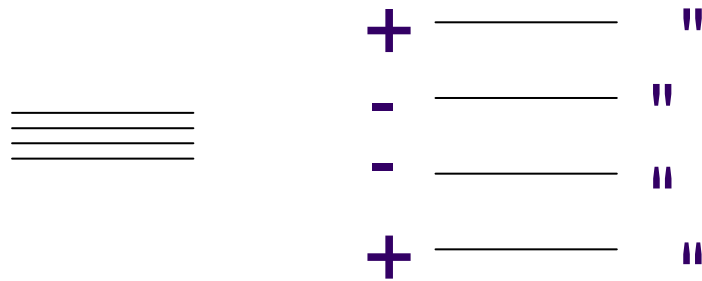
M=La;  
 $a=5.40484\text{\AA}$   
 $=3.8217 \times \sqrt{2}\text{\AA}$   
 $c=3.96185\text{\AA}$

M=Ca;  
 $a=5.36284\text{\AA}$   
 $=3.7921 \times \sqrt{2}\text{\AA}$   
 $c=3.69833\text{\AA}$



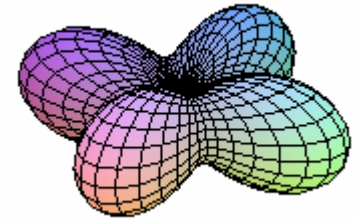
# Splitting of the $\Gamma_8$ level

- Magnetic field ( $H_z$ )



”+” orbital

” ”

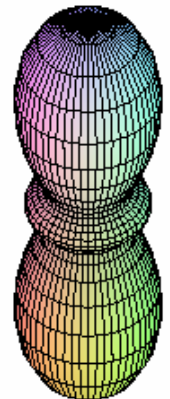


- Strain field ( $\epsilon_{zz}$ )



”-” orbital

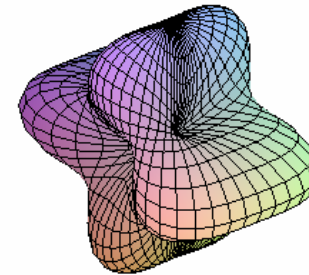
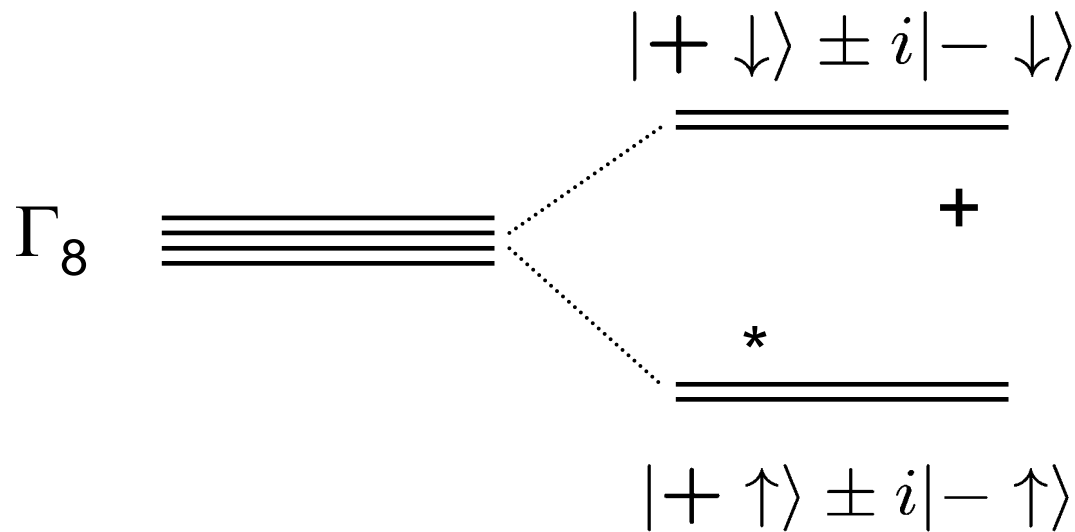
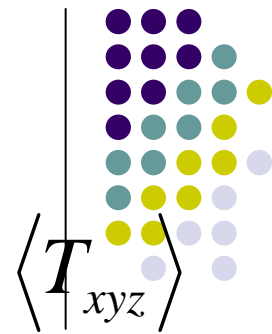
” ”



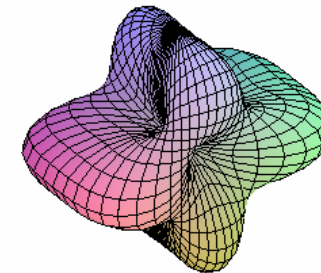
Broken T-reversal

+ unbroken orbital degeneracy

$T_{xyz}$  octupolar basis



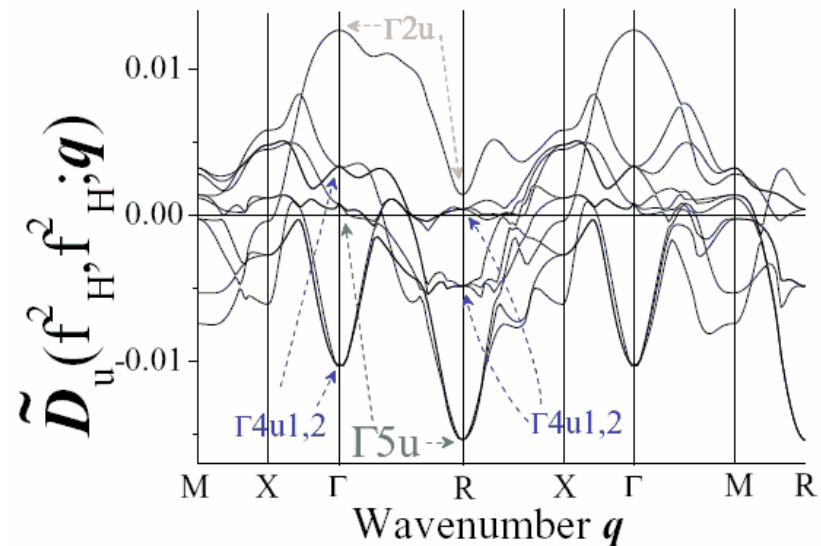
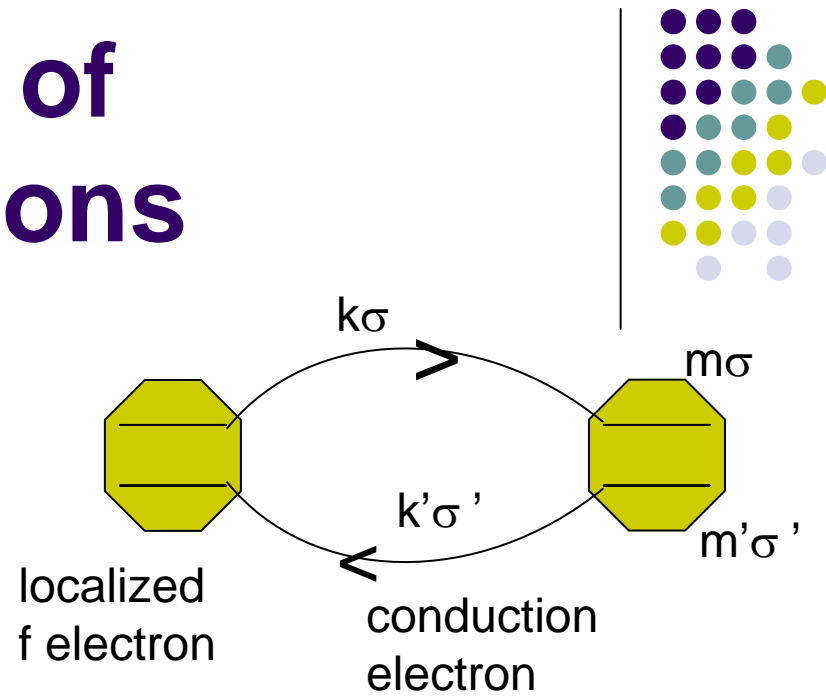
$$3\sqrt{3}/2$$



$$-3\sqrt{3}/2$$

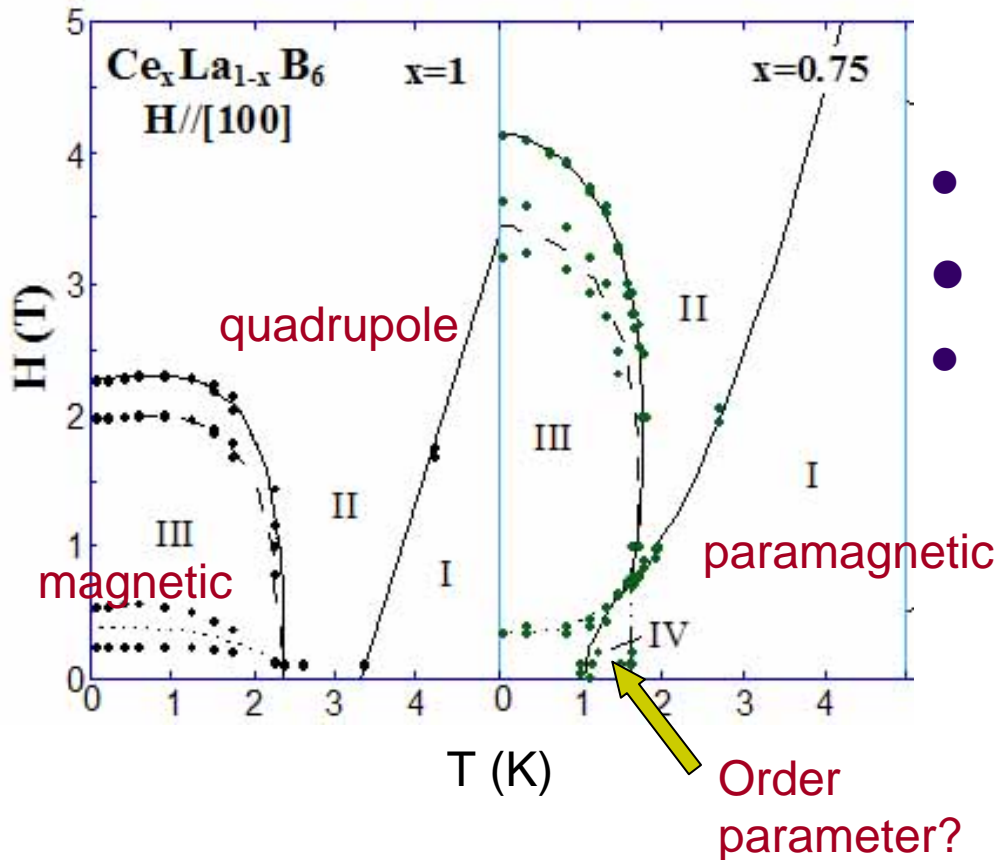
# Microscopic origin of multipolar interactions

- RKKY interaction
  - $(m\sigma) \Rightarrow (J, J_z) \Rightarrow (\Gamma\gamma)$   
spin-orbit      crystal field
- Octupolar interaction  
 $\Gamma_{5u}$  is the same order as the dipolar interaction  $\Gamma_{4u}$ 
  - G. Sakurai (Ph.D thesis)





# Strange ordered phase (phase ) in $\text{Ce}_x\text{La}_{1-x}\text{B}_6$



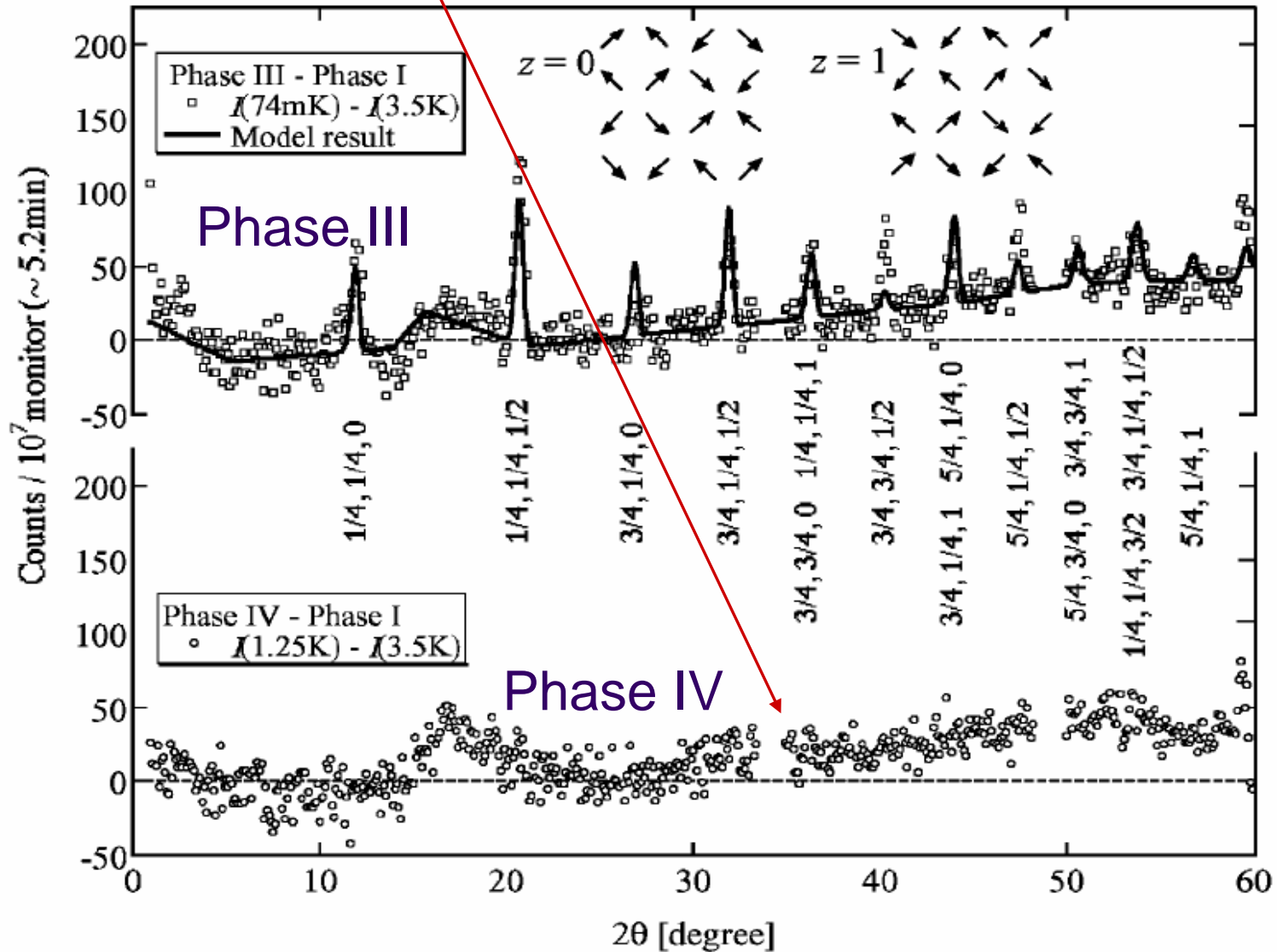
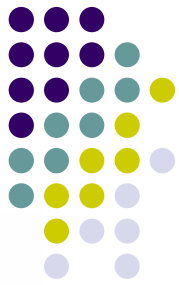
- No magnetic order by neutrons
- $\mu$ SR detects small internal fields
- Gigantic elastic anomaly and slight lattice distortion



Possible octupole order

Tayama *et al.* JPSJ (1997)

No magnetic Bragg peaks in phase IV



# Takagiwa et al

Journal of the Physical Society of Japan  
Vol. 71, No. 1, January, 2002, pp. 31–34  
©2002 The Physical Society of Japan

$\mu$ SR

$T_{IV} \gg 1.4 \text{ K}$

$H_{\text{int}} \gg 0.1 \text{ T}$

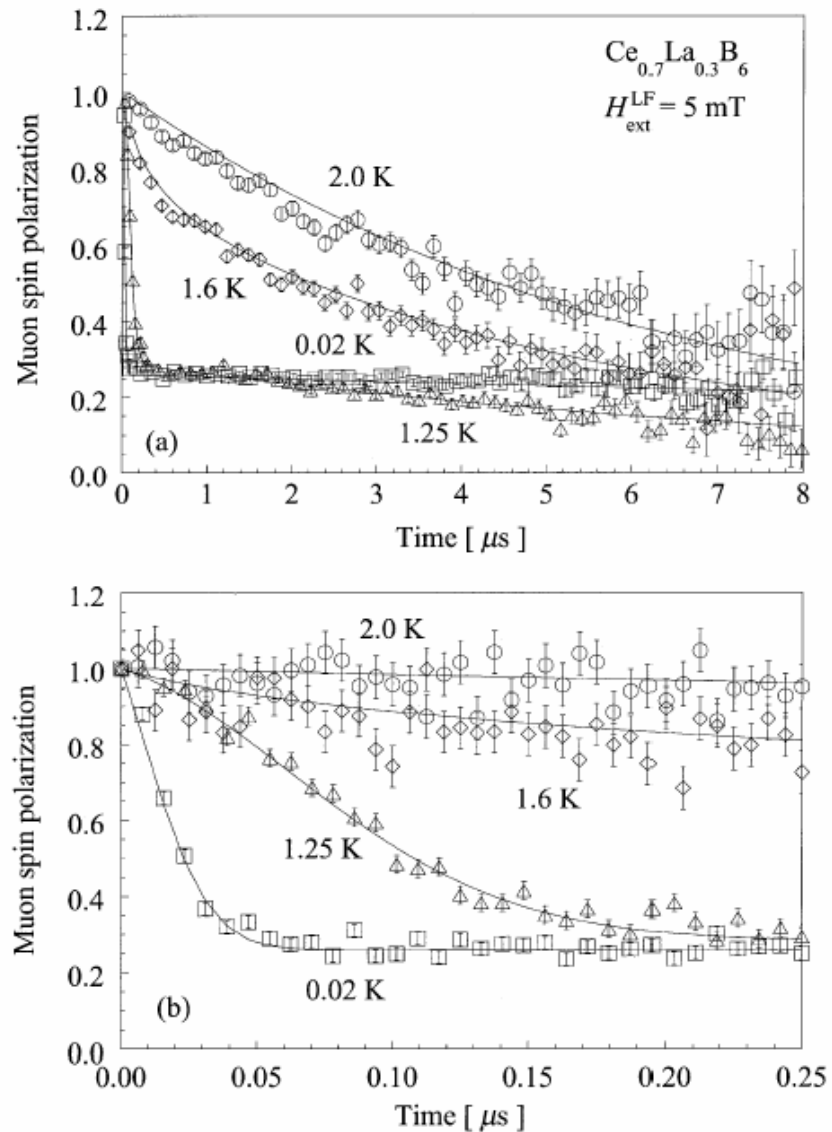
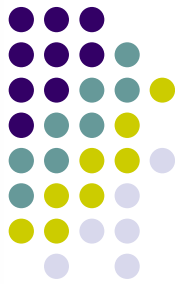
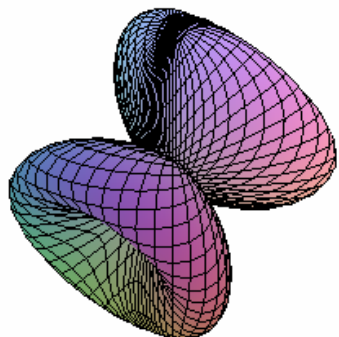
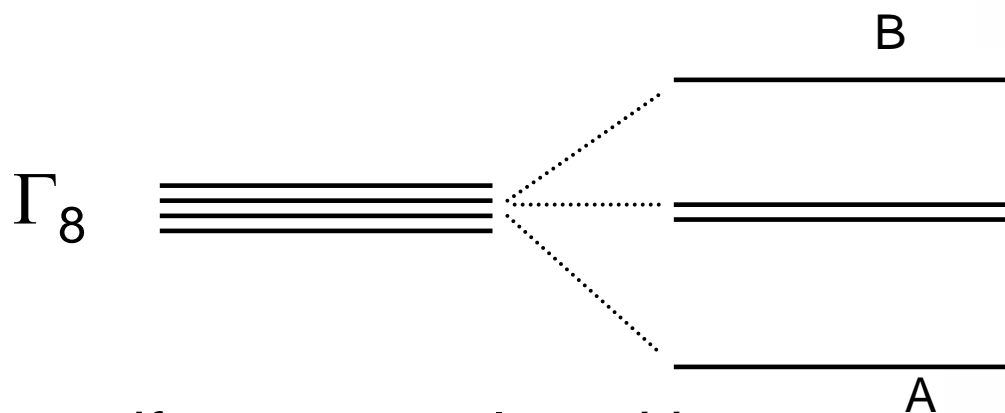


Fig. 1. (a) LF- $\mu$ SR time spectra (muon spin polarization) in  $\text{Ce}_{0.7}\text{La}_{0.3}\text{B}_6$  at temperatures of 2.0 K, 1.6 K (in phase I), 1.25 K and 0.02 K (in phase IV) and (b) an expanded view of (a). Solid curves are the results fitted by the function described in the text.

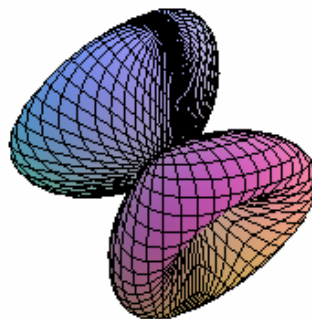
# Broken T-reversal and broken orbital degeneracy



$T_x^\beta + T_y^\beta + T_z^\beta$  octupolar basis

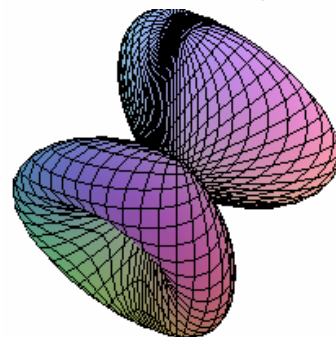


$3\sqrt{2}$



$\langle T_x^\beta + T_y^\beta + T_z^\beta \rangle$

0



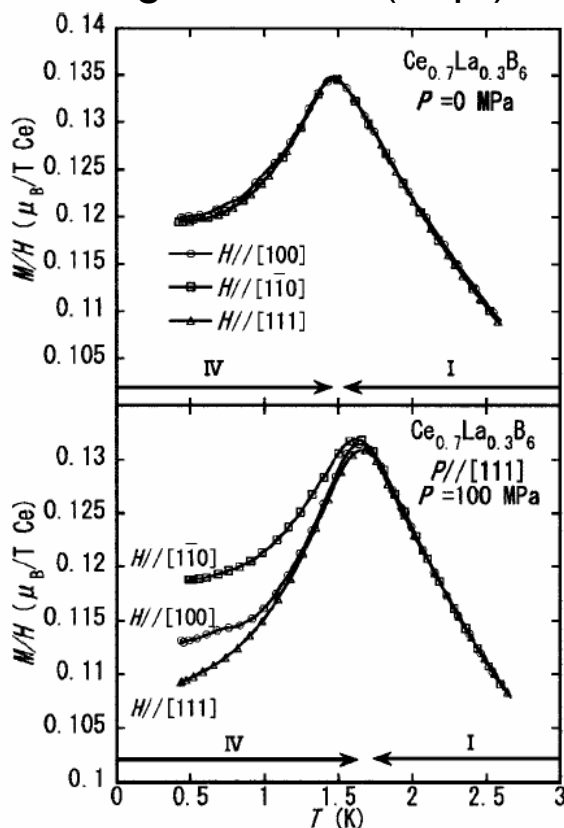
$-3\sqrt{2}$

antiferro-octupoles with  
 A,B sublattices )  
 ferro-quarupoles )  
 lattice distortion +  
 (1/2,1/2,1/2) Bragg peak

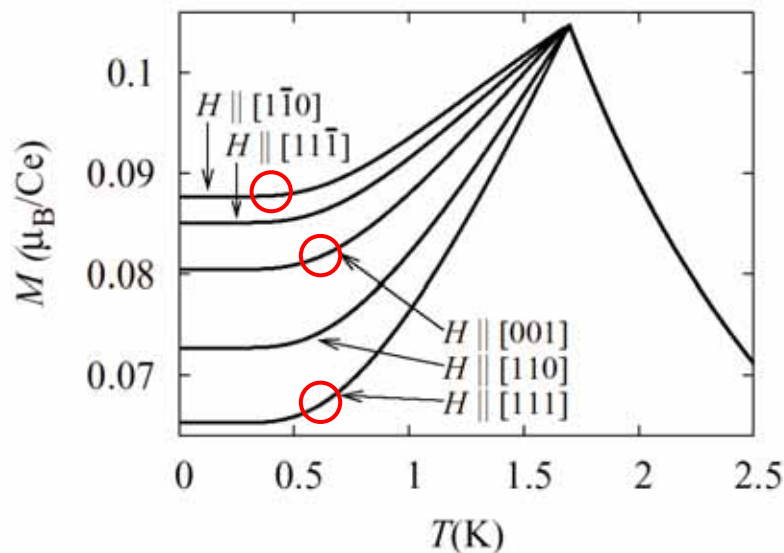
# Magnetic anisotropy in $\text{Ce}_{0.7}\text{La}_{0.3}\text{B}_6$ under uniaxial stress



Magnetization (Exp.)



Theoretical prediction

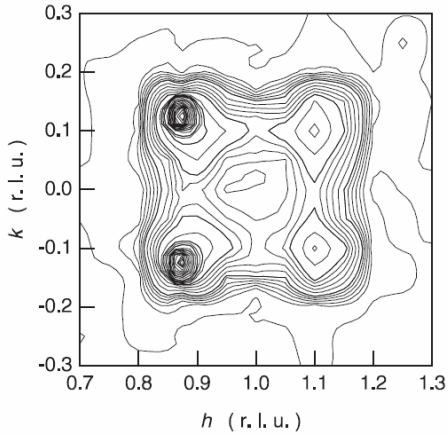
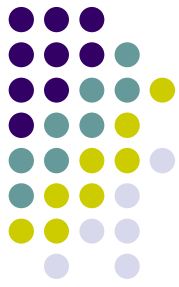


**Single octupole domain under uniaxial stress along [111]**

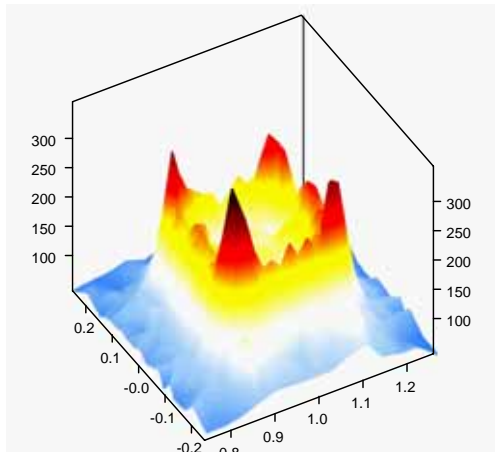
Theory: K. Kubo and Y. Kuramoto: J. Phys. Soc. Jpn. **73** (2004) 216.

Experiment: T. Morie *et al.*: J. Phys. Soc. Jpn. **73** (2004) 2381.

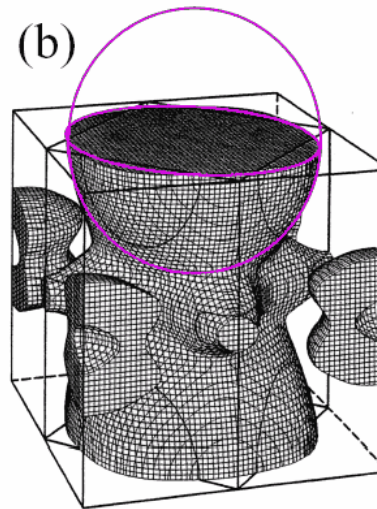
# Anomalous magnetic correlation in $\text{RB}_2\text{C}_2$ (R=Ho,Tb,Er) by Fermi surface effects?



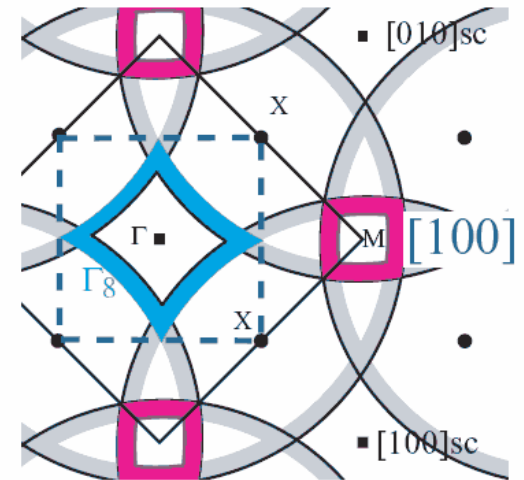
Neutron scattering: Ohoyama et al.  
Theory in progress: Sakurai et al.



$\text{HoB}_2\text{C}_2$

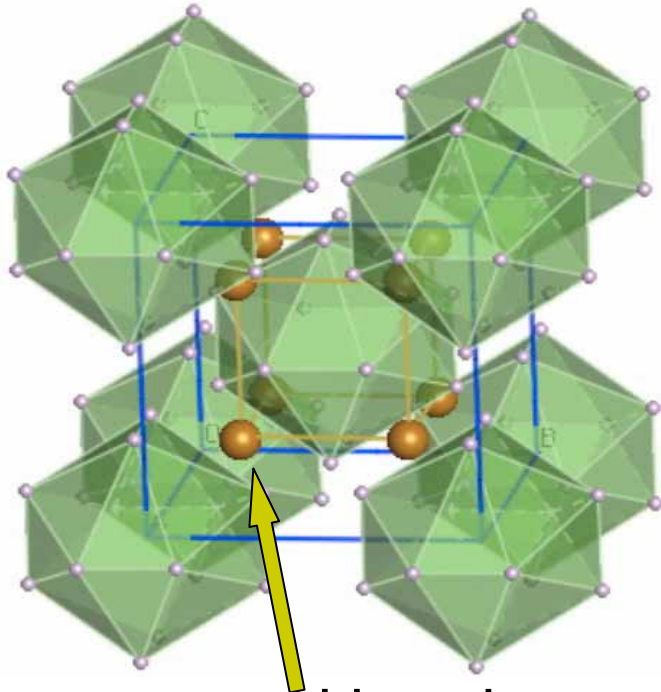
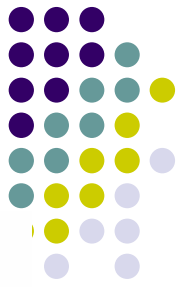


Fermi surface



Kohn anomaly

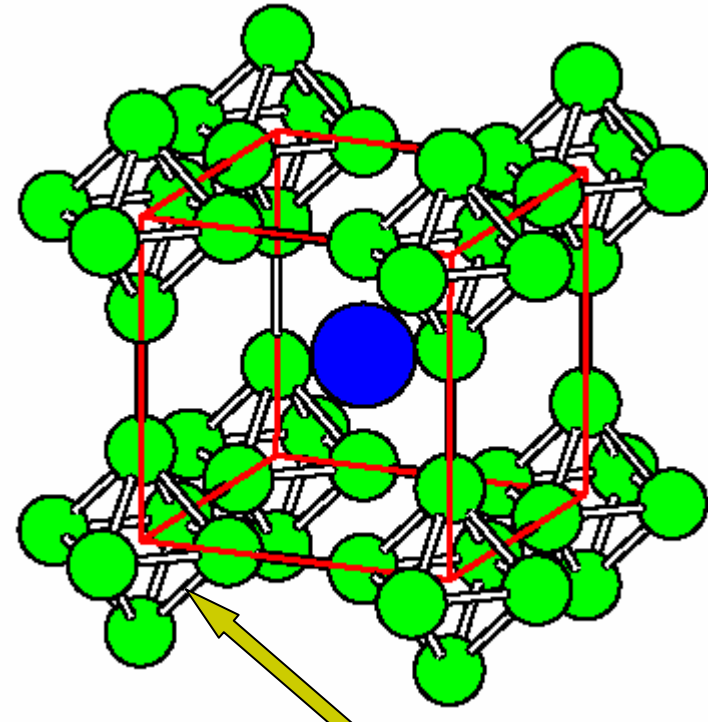
# Clathrate-like structures



positive charge

Pr skutterudite:  $\text{PrT}_4\text{X}_{12}$

ground state: **nondegenerate**



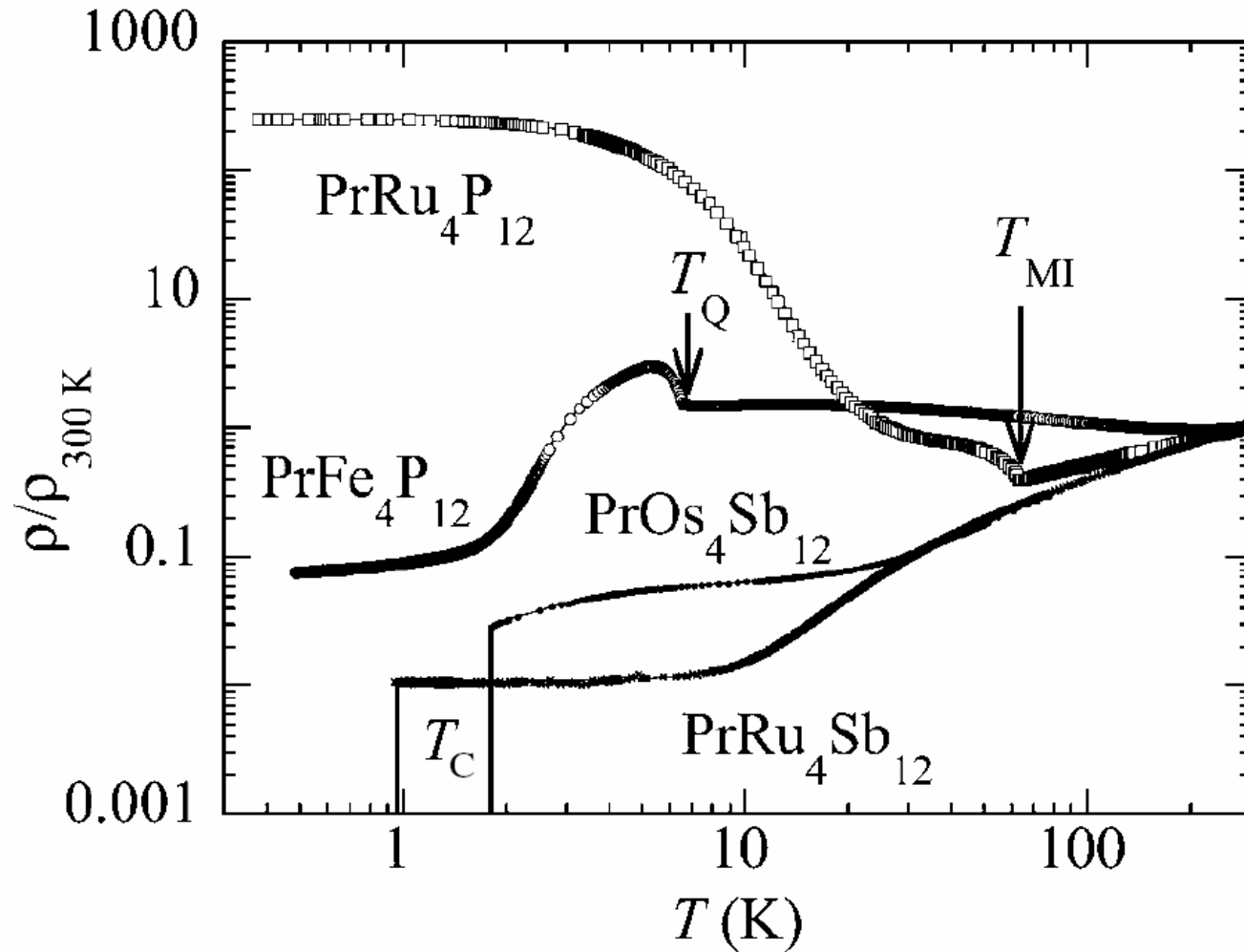
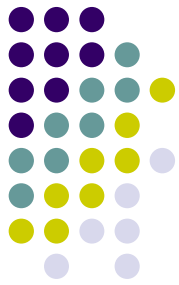
negative charge

$\text{PrB}_6$

ground state: **triplet**

# Resistivity $\rho(T)$ in Pr skutterudites

H. Sato et al.: J. Phys.: Condens. Matter 15 (2003) S2063–S2070







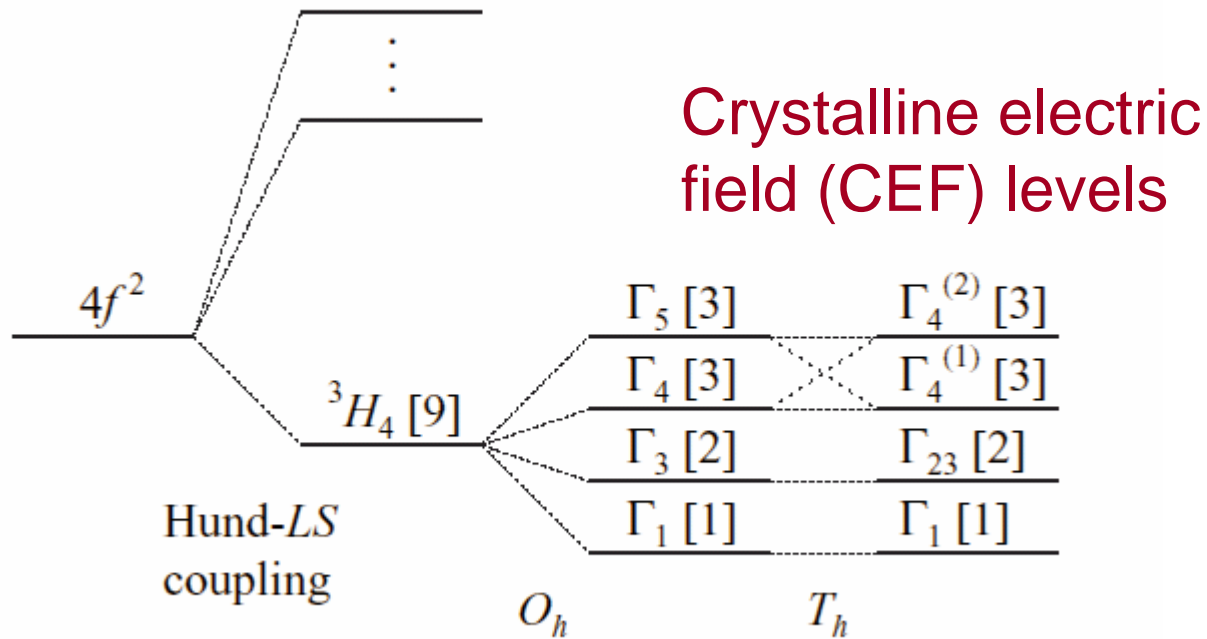
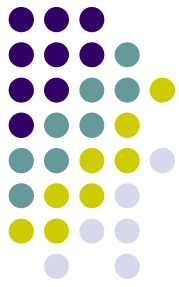
# Pr skutterudites: quantum zoo

- Phase diagram with **multiple orders**
  - $\text{PrFe}_4\text{P}_{12}$  => Antiferro-quadrupole (AFQ) order
  - $\text{PrOs}_4\text{Sb}_{12}$  => AFQ and superconductivity
- **Metal insulator transition** and temperature-dependent f-electron levels
  - $\text{PrRu}_4\text{P}_{12}$
- **Kondo effect** in resistivity and magnetic response
  - $\text{PrFe}_4\text{P}_{12}$



controlling parameter ? ) p-f hybridization !

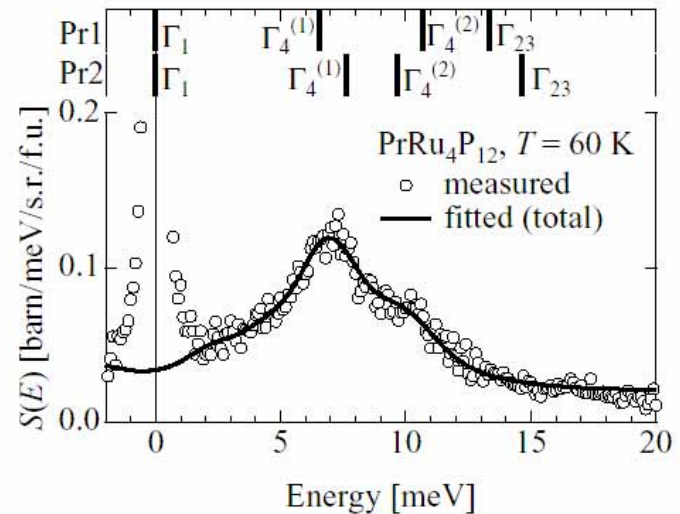
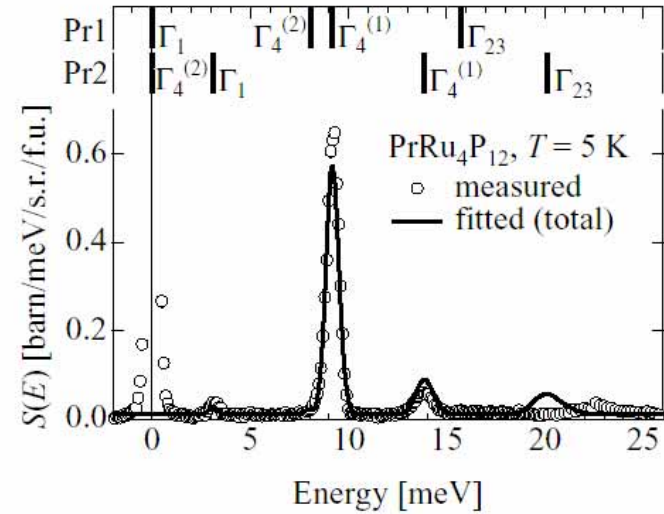
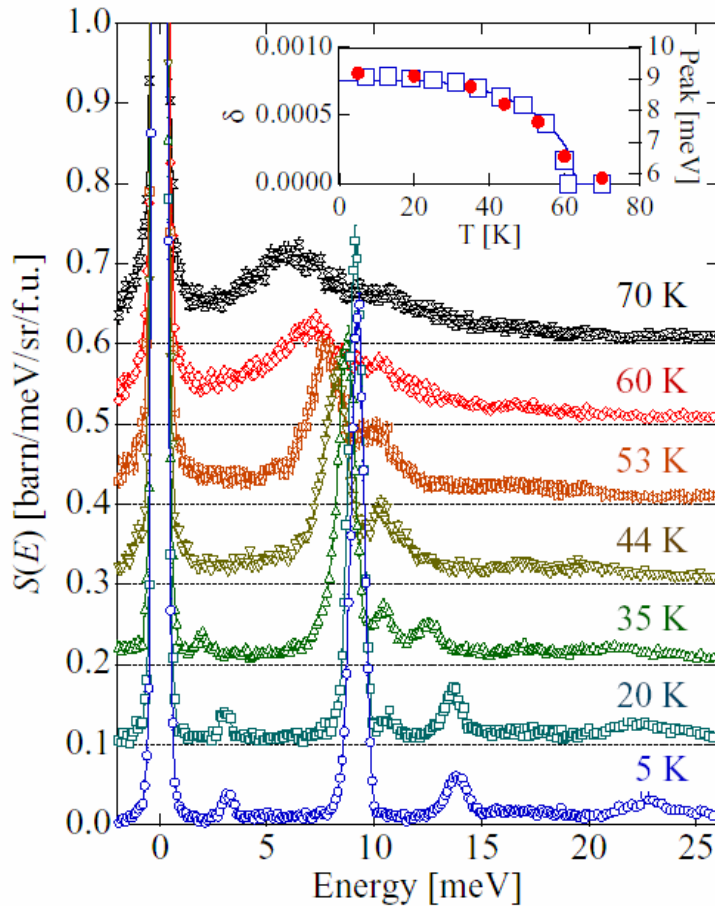
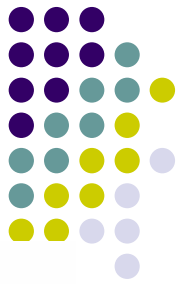
# J=4 CEF levels in point-group symmetries $O_h$ and $T_h$

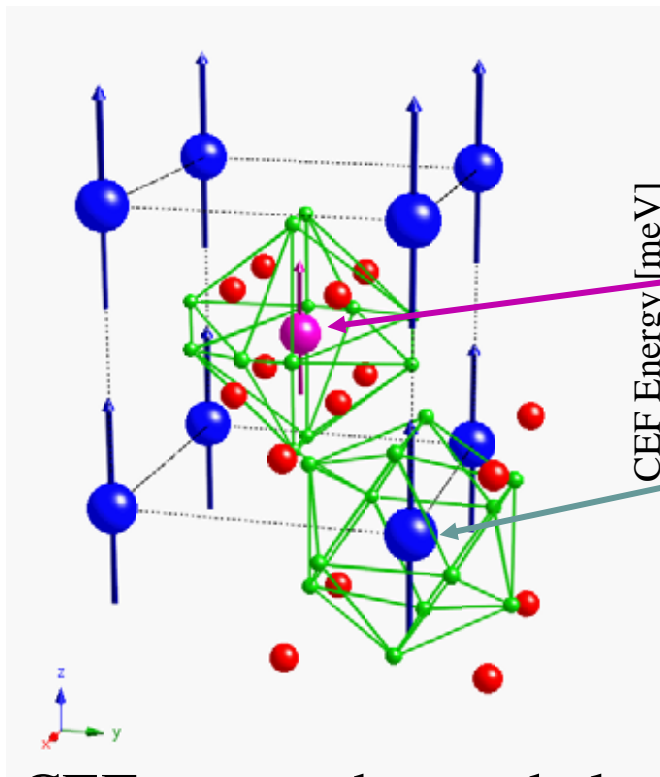


$L=5, S=1,$   
 $J=4$

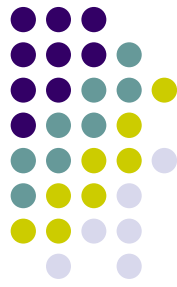
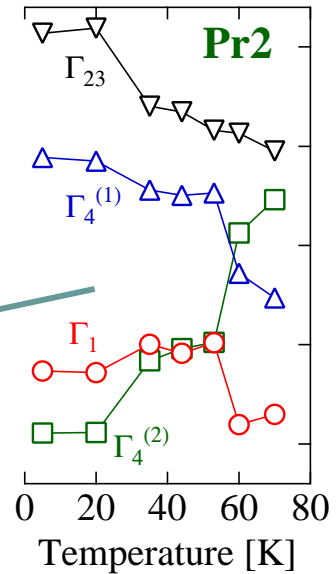
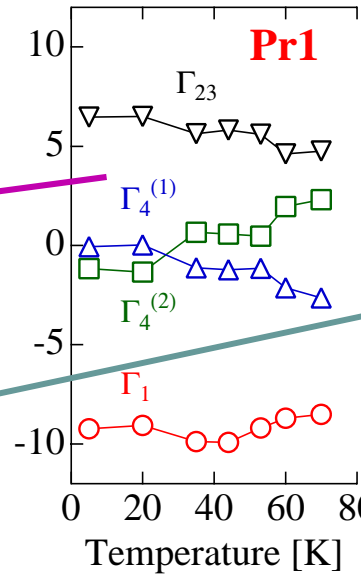
# Neutron scattering of $\text{PrRu}_4\text{P}_{12}$

(Iwasa et al.: 2004)



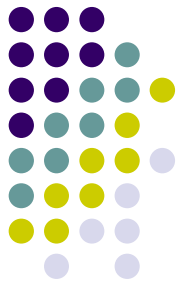


CEF Energy [meV]

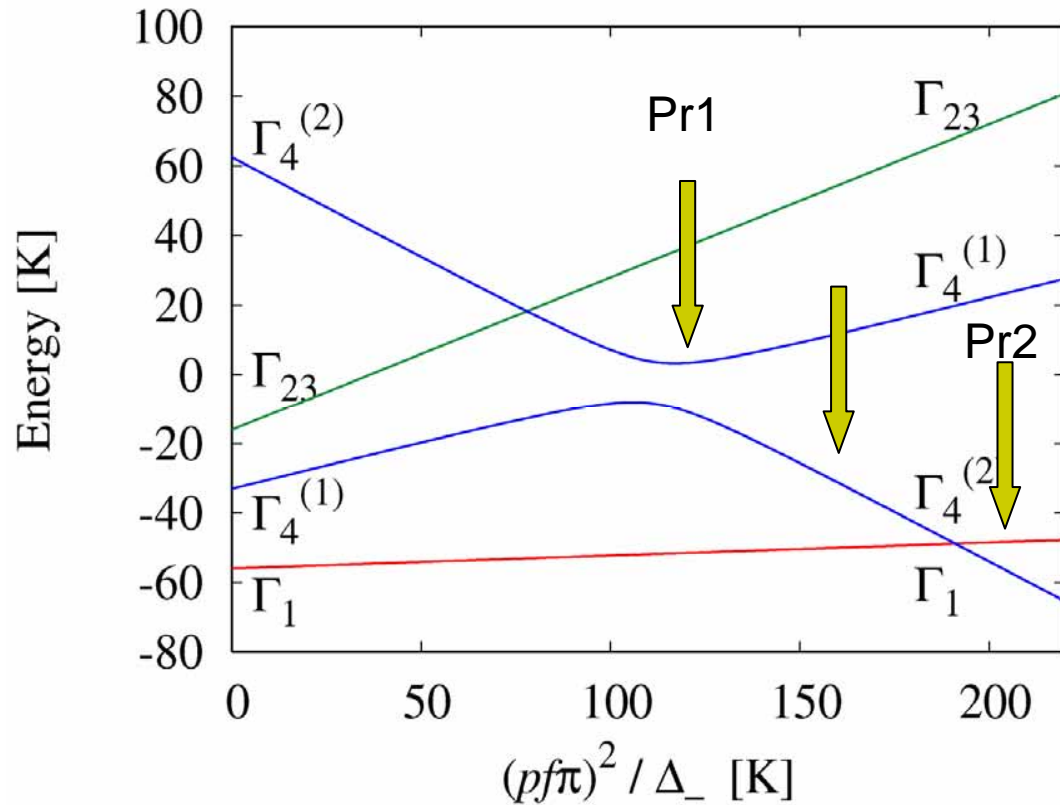


- The CEF states change below the M-I transition of  $\text{PrRu}_4\text{P}_{12}$ , and superlattice crystal-structure is observed.
- Two inequivalent CEF schemes for the Pr-ion states below  $T_{\text{M-I}}$ .
  - $\text{Pr}_1$  and Ru ions move closer :  $\Gamma_1 - \Gamma_4^{(2)} - \Gamma_4^{(1)} - \Gamma_{23}$
  - $\text{Pr}_2$  and Ru move farther:  $\Gamma_4^{(2)} - \Gamma_1 - \Gamma_4^{(1)} - \Gamma_{23}$
- These phenomena are explained by **the strong hybridization (p-f mixing) effect.**

# CEF levels: point charge + hybridization



point charge only



$$1/\Delta_- = 1/\Delta_3 - 1/\Delta_1$$

- Hybridization ( $pf\pi$ ) only is relevant to the  $a_u$  band.

- Point-charge parameters:  
 $Z_t=2, Z_p=-11/12$

PrOs<sub>4</sub>Sb<sub>12</sub>?

$$(pf\pi)^2 / \Delta_- = 190\text{K}$$

$$\rightarrow \Delta_1 \simeq 5\text{eV}, \Delta_3 \simeq 3\text{eV},$$

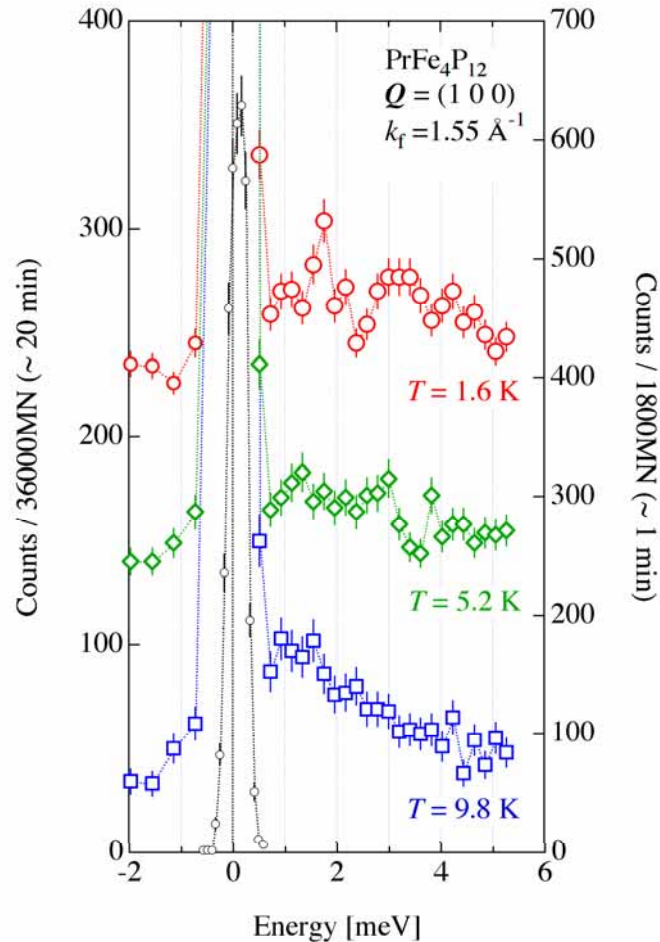
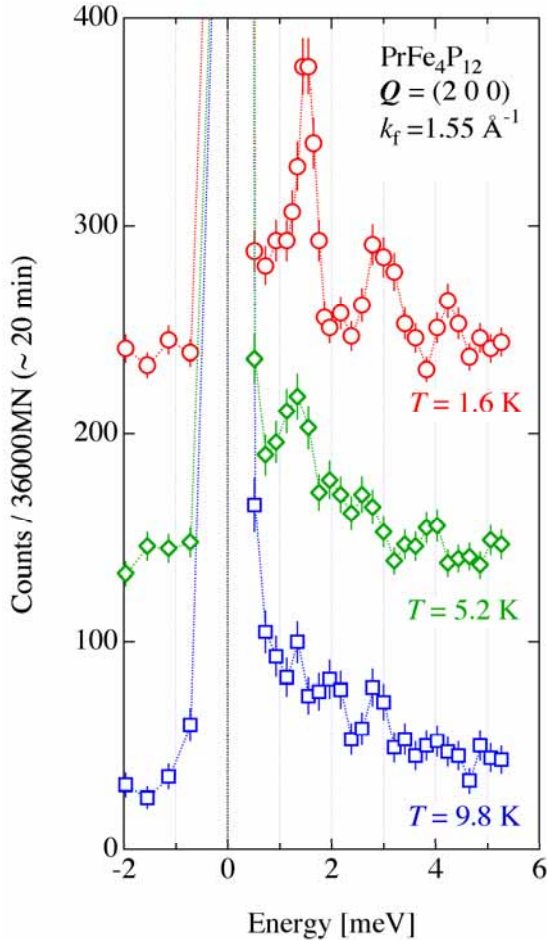
$$(pf\pi) \simeq 0.35\text{eV}$$

$f^1$  and  $f^3$  intermediate states compete to make CEF levels

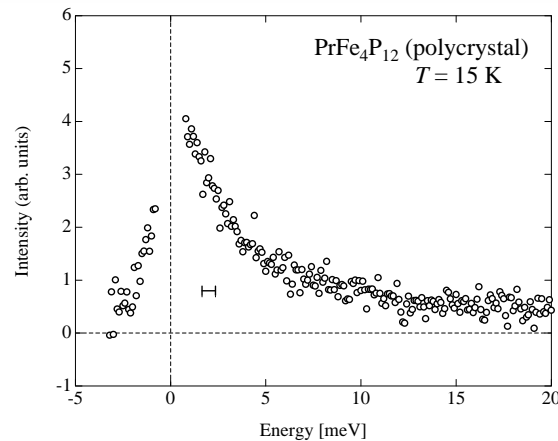
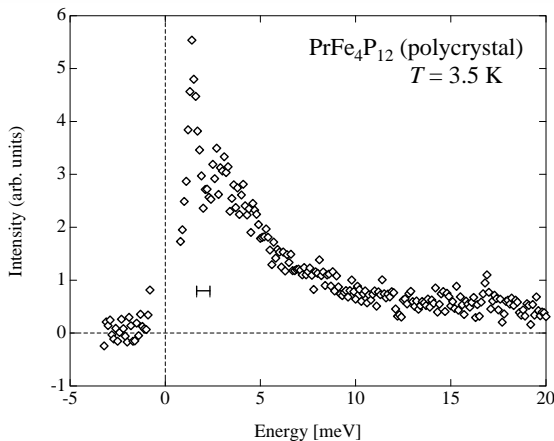
$\text{PrFe}_4\text{P}_{12}$   
Iwasa et al  
(2003)

Peak position:  
T-independent  
=>  
CEF excitation!  
Why so broad?

single crystal



polycrystal



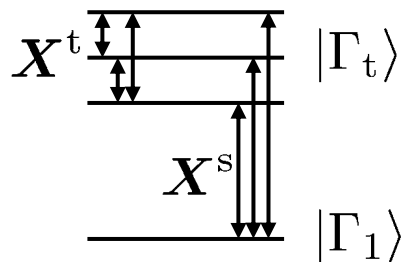
# Effective exchange interaction in the singlet-triplet system



$$\begin{aligned}
 H_{s-t} &= \epsilon_t P_t + \epsilon_s P_s + (I_t \mathbf{X}^t + I_s \mathbf{X}^s) \cdot \mathbf{s}_c \\
 &= \Delta_{\text{CEF}} \mathbf{S}_1 \cdot \mathbf{S}_2 + (J_1 \mathbf{S}_1 + J_2 \mathbf{S}_2) \cdot \mathbf{s}_c
 \end{aligned}$$

$$(\Delta_{\text{CEF}} = \epsilon_t - \epsilon_s)$$

Kondo effect occurs if  $J_i > 0$  (antiferromagnetic exchange)



$$\mathbf{X}^t = \mathbf{S}_1 + \mathbf{S}_2$$

$$\mathbf{X}^s = \mathbf{S}_1 - \mathbf{S}_2$$

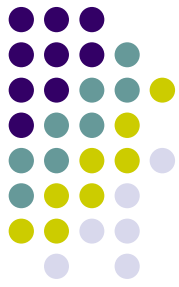
$\mathbf{S}_1, \mathbf{S}_2$ : pseudo-spin

$$|\Gamma_1\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

$$|\Gamma_{t,+}\rangle = |\uparrow\uparrow\rangle$$

$$|\Gamma_{t,0}\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$$

$$|\Gamma_{t,-}\rangle = |\downarrow\downarrow\rangle$$



# Effective exchange with conduction electrons

Triplet wave functions in the point group  $T_h$  ( $m=0, \pm$ )

$$|\Gamma_t, m\rangle = \sqrt{w}|\Gamma_4, m\rangle + \sqrt{1-w}|\Gamma_5, m\rangle$$

Effective interaction

$$H_{\text{eff}} = PH_{\text{hyb}}(E - H_0)^{-1}QH_{\text{hyb}}P$$

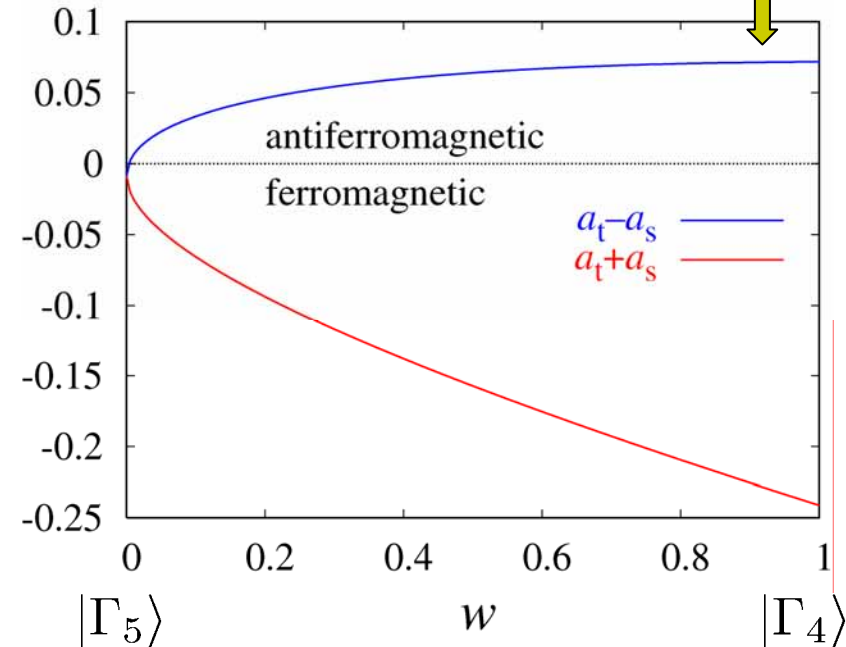
$$H_{\text{hyb}} = V_{2u} \sum_{\sigma} f_{\sigma}^{\dagger} c_{\sigma} + \text{h.c.}$$

$$J_1 = (a_t + a_s)V_{2u}^2/\Delta$$

$$J_2 = (a_t - a_s)V_{2u}^2/\Delta$$

$$(1/\Delta = 1/\Delta_1 + 1/\Delta_3)$$

Kondo effect can take place!



Small coupling constants in the case of  $\Gamma_1$ - $\Gamma_5$   
 Large antiferro-coupling in the case of  $\Gamma_1$ - $\Gamma_4$

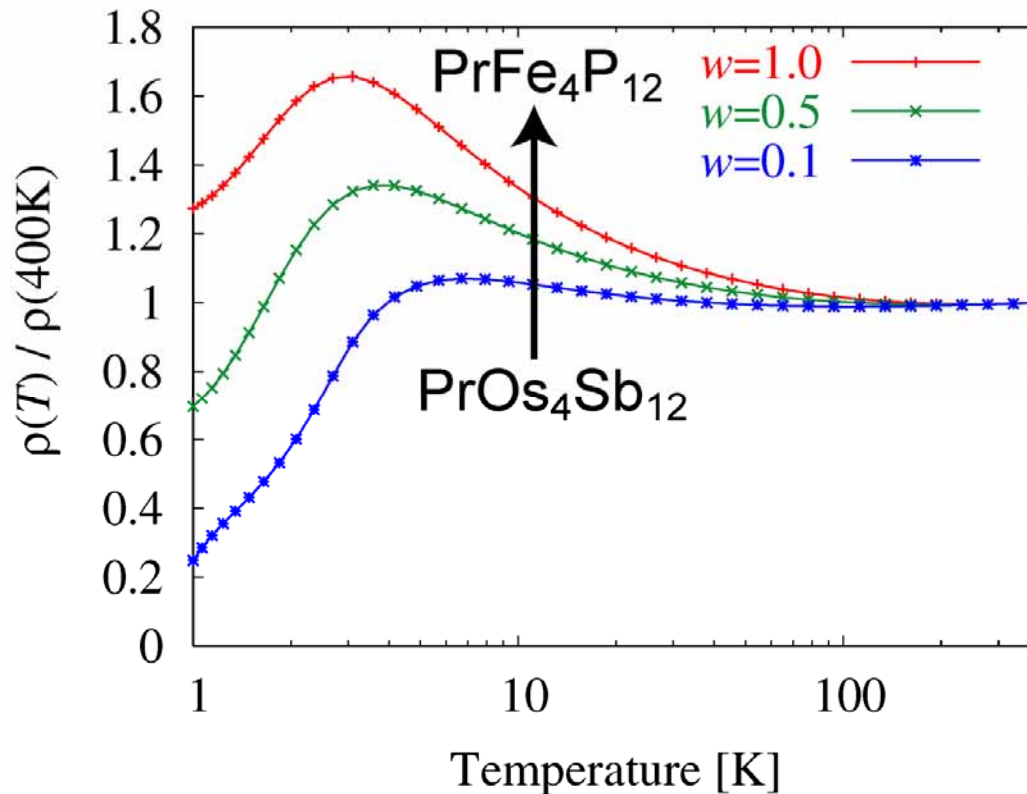
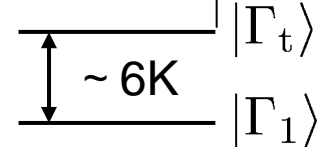


# Resistivity in the CEF pseudo-quartet model by the Non-Crossing App. (NCA) ...Otsuki

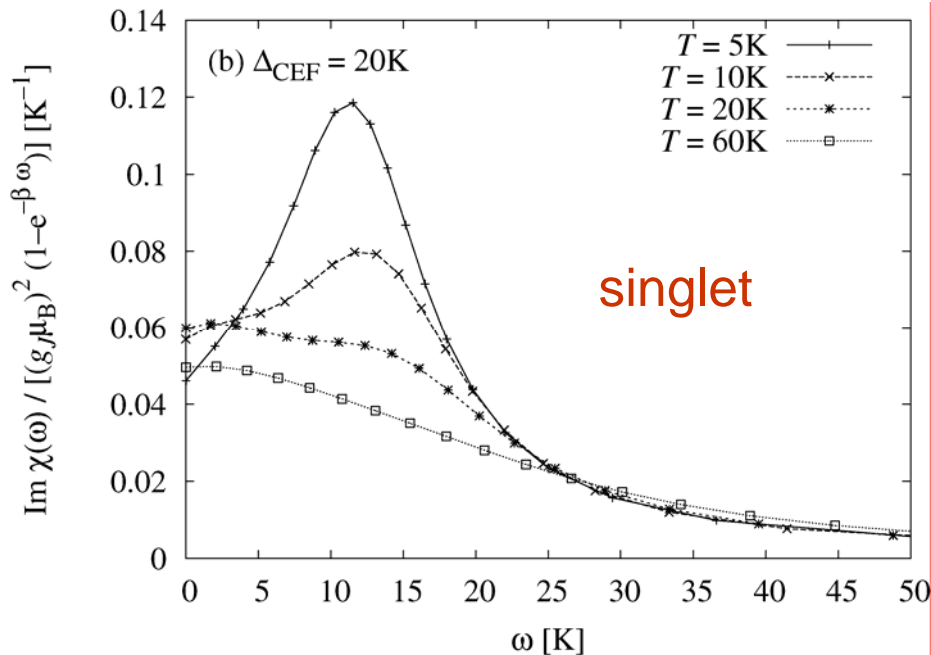
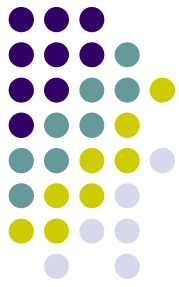


$$H_{\text{int}} = (I_t \mathbf{X}^t + I_s \mathbf{X}^s) \cdot \mathbf{s}_c$$

$$|\Gamma_t, m\rangle = \sqrt{w} |\Gamma_4, m\rangle + \sqrt{1-w} |\Gamma_5, m\rangle$$



# Neutron scattering spectrum in the singlet-triplet model by the NCA (Otsuki)



$$T \gtrsim \Delta_{\text{CEF}}:$$

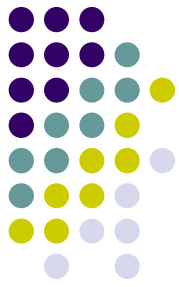
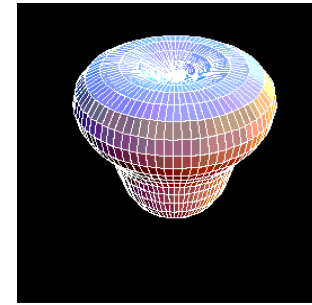
Only quasi-elastic peak

$$T \lesssim \Delta_{\text{CEF}}:$$

Inelastic peak develops as temperature decreases.

$$J_1 \rho_c = 0.2, \quad J_2 \rho_c = 0, \quad \Delta_{\text{CEF}} = 20\text{K}$$

# Summary and outlook



- Octupoles can order **without dipole order** in  $\text{Ce}_{1-x}\text{La}_x\text{B}_6$ 
  - theoretical prediction & exp. confirmation provided
- Peculiar magnetic correlation in  $\text{RB}_2\text{C}_2$  probed by neutron scattering
  - to be explained in terms of the **Kohn anomaly**
- Anomalous local excitation and distortion in Pr skutterudites by p-f hybridization
  - **to be further developed by experiment+theory cooperation**