Single-point condensation phenomena for a four-dimensional biharmonic semilinear problem

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We study the asymptotic behavior of least energy solutions to the following fourth order elliptic problem (E_p):

$$(E_p)\begin{cases} \Delta^2 u = u^p \quad \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \\ u|_{\partial\Omega} = \Delta u|_{\partial\Omega} = 0 \end{cases}$$

as p gets large, where Ω is a smooth bounded domain in $\mathbf{R}^4$.

In our earlier work, we have shown that the least energy solutions remain bounded uniformly in p and they have one or two "peaks" away form the boundary.

In this talk, following the arguments in Adimurthi-Grossi and Lin-Wei, we will obtain more sharper estimates of the upper bound of the least energy solutions and prove that the least energy solutions must develop single-point spiky pattern, under the assumption that the domain is convex.

More precisely, we have

Theorem. *Assume Ω is a smooth bounded convex domain in $\mathbf{R}^4$ and u_p is the least energy solutions to (E_p). Then we have*

$$1 \le \liminf_{p\to\infty} \|u_p\|_{L^\infty(\Omega)} \le \limsup_{p\to\infty} \|u_p\|_{L^\infty(\Omega)} \le \sqrt{e}.$$

Furthermore, set $w_p := u_p/(\int_\Omega u_p^p dx)$. Then for any sequence w_{p_n} of w_p with $p_n \to \infty$, there exists a subsequence such that the blow up set S of this subsequence satisfies $S = \{x_0\}$ (one point blow up) for $x_0 \in \Omega$, and

(1) $f_{p_n}(x) := \frac{u_{p_n}^{p_n}(x)}{\int_\Omega u_{p_n}^{p_n} dx} \overset{*}{\rightharpoonup} \delta_{x_0}$ *in the sense of Radon measures of Ω.*

(2) $w_{p_n} \to G_4(\cdot, x_0)$ *in $C^4_{loc}(\overline{\Omega} \setminus \{x_0\})$ where $G_4(x,y)$ denotes the Green function of Δ^2 under the Navier boundary condition.*

(3) *x_0 is a critical point of the Robin function $R_4(x) = H_4(x,x)$, where $H_4(x,y) := G_4(x,y) + \frac{1}{8\pi^2}\log|x-y|$ denotes the regular part of G_4.*

We conjecture that the blow up point x_0 may be a maximum point of the Robin function R_4. This is a future work for us.

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