The 21 Century COE Project
Exploring New Science by Bridging Particle-Matter Hierarchy

Short-term Foreign Researchers

Research Report

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Your Stay Period in Japan: From 23/Feb/2003 to 19/March/2003
Title of Research in Japan:
Noncommutative Geometry and Its Application to Low-Dimensional Physics

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Please write a research report of one or more pages and submit it with this cover to your host researcher till the end of this March.

Research Report by G. Tsitsishvili

The research was conducted in collaboration with Professor Z.F. Ezawa at the Department of Physics of Tohoku University. The topics under consideration concerned the field theoretic studies of quantum Hall systems from a viewpoint of *noncommutative geometry*. Namely, the hosting and visiting sides recently developed a formalism for description of QH systems [1] based on noncommutative geometry, and the current activities represent the application of this formalism to the bilayer QH system at the filling $\nu = 1$.

The given considerations have been started about a year ago and have been taking relatively big amount of time, since the collaborative studies used to be carried out using the e-mail communications. For the same reason, some significant features of the issue couldn't be traced out at all.

In this respect, the visit was undoubtedly helpful, since it gave the possibility to hold live discussions around the subject. Several key points being missed in previous studies have been revealed. The following two features should be pointed out.

- Requiring the general skyrmion-type behaviour of the average fields, we used the reduced variables constructed in [1] and reproduced the special type of Fock states which had been obtained in [2] from different point of view. Using the anzatz proposed in [2] we managed to sum up the involved series and expressed all the relevant physical quantities in terms of known (Kummer) functions. This allowed to analyze the asymptotic characteristics of the skyrmion fields which may arise in quantum Hall systems. In this consideration it was found out that the standard form of skyrmions usually employed in relevant studies are valid only asymptotically, while the bulk properties of QH skyrmions are essentially of noncommutative nature.

- Using the general microscopic form of N-component QH skyrmions, we showed that the corresponding classical field is normalized noncommutatively (via the Moyal product) as

$$S_a(\boldsymbol{x}) \star S_a(\boldsymbol{x}) + \frac{1}{2N}\rho(\boldsymbol{x}) \star \rho(\boldsymbol{x}) = \frac{1}{4\pi\ell^2}\rho(\boldsymbol{x})$$

  where S_a and ρ are the spin and particle densities, respectively. This relation which hasn't been known earlier, can be regarded as a manifestation of the noncommutative properties of QH systems at the level of effective theories.

- Using the anzatz proposed in [2], we showed that the corresponding Coulomb theory is free of infrared divergences. However, if the Zeeman terms are involved, we found that the anzatz of [2] fails, since the Zeeman energy becomes divergent. As a way out of this problem, we figured out the alternative anzatz which is free of divergences (even with Zeeman term included) since it leads to the exponential suppression of the fields, while the anzatz of [2] leads to the polynomial one. The corresponding calculations are intimately related with noncommutative effects and therefore we consider the relevant analysis as a separate future project.

The basic results are included in the revised version of cond-mat/0311406, which is now ready to be submitted for publication in *Phys. Rev.* **B**.

[1] Z.F. Ezawa, G. Tsitsishvili and K. Hasebe, *Phys. Rev.* **B67** (2003) 125314.
[2] H.A. Fertig, L. Brey, R. Cote and A.H. MacDonald, *Phys. Rev.* **B50** (1994) 11018.