The 21 Century COE Project
Exploring New Science by Bridging Particle-Matter Hierarchy

Short-term Foreign Researchers

Research Report

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Your Stay Period in Japan: From 2003.12.1 to 2003.12.13

Title of Research in Japan:

Qualitative studies of nonlinear parabolic equations

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Please write a research report of one or more pages and submit it with this cover to your host researcher till the end of this March.

Asymptotic Behavior of Solutions to the Cauchy Problem for the Equation $u_t = (\log u)_{xx} + u$

Peter TAKÁČ and Eiji Yanagida

We investigated the long-time asymptotic behavior of (positive) weak solutions $u : \mathbb{R}\times\mathbb{R}_+ \to (0,\infty)$ to the Cauchy Problem

$$\text{(CP)} \qquad \begin{cases} \partial_t u = \partial_x^2(\log u) + u & \text{for } (x,t) \in \mathbb{R}\times(0,\infty); \\ \lim\limits_{x\to-\infty} \partial_x(\log u) = -V_1 \ \text{ and } \ \lim\limits_{x\to+\infty} \partial_x(\log u) = -V_2 & \text{for } t \in (0,\infty); \\ u(x,0) = u_0(x) & \text{for } x \in \mathbb{R}. \end{cases}$$

The constants $V_{1,2}$ are given such that $-\infty < V_1 < 0 < V_2 < \infty$. We consider only (positive) weak solutions satisfying $\int_{-\infty}^{\infty} u(x,t)\,\mathrm{d}x < \infty$ and $\operatorname{ess\,sup}\limits_{x\in\mathbb{R}} |\partial_x(\log u)(x,t)| < \infty$ at all times $t \geq 0$. We discover an interesting fact that the dynamics of this problem is fully described by that of the special case when the total mass is conserved, i.e., $\int_{-\infty}^{\infty} u(x,t)\,\mathrm{d}x = V_2 - V_1$ for each $t \geq 0$. In this case we show that, as $t \to \infty$, such a solution must approach a travelling pulse in the norm of $L^1(\mathbb{R})$. If the total mass is smaller, we establish extinction of the solution in finite time, whereas if it is larger, we obtain exponential growth of the solution as time goes to infinity. In the last two cases, we provide an easy transformation to the special case with the total mass conserved.